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VOLUME L NUMBER 1

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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JULY 1919

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THE
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VOLUME L JULY 1919 NUMBER 1

THE PHYSICAL INTERPRETATION OF ALBEDO, II
SATURN'S RINGS

By LOUIS BELL

Saturn and its rings form easily the most interesting and spectacular feature of the planetary system. The rings from the time they puzzled Galileo to the present have been a favorite subject of study and speculation. Their general character was finally settled by the brilliant dynamical investigation of Maxwell and the no less brilliant spectroscopic study of Keeler, but even these left unsolved the mystery of their intimate structure and the curious phenomena from time to time recorded of them. It is the purpose of this paper to see what added information may be extorted from a study of their reflecting power.

Speaking broadly, Saturn is in itself a rather uninteresting yellowish ball with a bright and broad equatorial band shading off somewhat variously from time to time into rather dull polar caps. Its geometrical albedo is very close to 0.42, and consequently its albedo as a body following Lambert's law is very nearly 0.63. These absolute values play only a subsidiary part in the investigation which follows.

The chief feature here to be considered is that the equatorial belt is notably brighter than the rest of the planet. To account for its proportionate effect on the general brilliancy of the globe

the geometrical albedo must be at least 0.50, indicating a general reflectivity as a diffusing body of about 0.75. In other words, this particular region has approximately the reflectivity of a bright cloud. No proper photometric investigation of the relation of this to the brilliancy of the rings has yet been made. But most observers agree that the outer and brighter part of the *B* ring is both materially brighter and whiter than the bright equatorial belt. Toward the inner edge of *B* this brilliancy is somewhat reduced. The *A* ring is notably less brilliant than the *B* ring, matching the darker portions of the ball, although perceptibly brighter than these toward its inner edge near Cassini's division. The *C* ring is, as every observer knows, ordinarily extremely faint, with a brightness certainly not over a few hundredths of that of the bright part of the *B* ring. Compared with this low figure the *A* ring is immensely bright, although the gross reflectivity of its diffusing material can scarcely exceed 0.3. One has, therefore, in examining the ring system, to account for albedo varying over an enormously great range, from a point well up toward the reflecting limit of known materials down to a faintness comparable with that of black paper.

That the rings are of good diffusing material is clear from their behavior under varying angles of incidence. It is therefore pertinent to examine the limits of reflectivity of diffusing matter, since whatever light the rings return they unquestionably receive either directly from the sun or in meager measure indirectly from the planet. The old suggestion that they may shine in virtue of some occult electrical phenomena may be put aside with the intimation that light from such a source would show a bright-line spectrum if it were really brilliant enough to amount to anything as compared with the reflected light.

It is well known that all substances when finely powdered reflect light diffusely, and sometimes remarkably well. Even some minerals of very dark color show a whitish streak when abrasion powders the surface, and, very generally, colored substances when more and more finely powdered grow lighter in color. This is due chiefly to the fact that the color of any body, be it powdered or solid, is produced by multiple selective reflection from particle to particle in the surface. The finer and more smoothly laid the

particles the less coloration, and also the less modification of hue by shadowing, facts easily seen when one presses flat with a piece of glass a loose surface of dry pigment, like ochre. Hence a smooth flat surface of compressed fine powder shows higher total reflectivity than the same material in any looser aggregation, as Nutting[1] has shown.

Finally, when the dimensions of the particle are small compared with the wave-length of light, the well-known phenomenon of selective scattering appears, as Rayleigh[2] long ago showed, which is as the inverse fourth power of the wave-length, hence highly selective for the blue, violet, and ultra-violet.

When, therefore, a beam of white light strikes a surface of optical dust, meaning thereby matter so finely divided that its reflecting power must be considered with respect to the wave-length of light, whether in free space or confined upon a surface, the amount and character of the light diffusely reflected depend both on the material and on its state of subdivision. A minute amount of the incident energy is spent in producing acceleration of the particles, a portion is selectively absorbed in single or multiple reflections, and if the dust is fine enough a considerable part of the beam is selectively scattered. No substance is wholly non-selective in its reflection, and when finely subdivided every substance is selective in its scattering and always in a direction to lower its luminosity through the λ^{-4} law. Even when the particles are of dimensions to scatter all visible wave-lengths very perceptibly and so produce a whitish ensemble, there is still lack of luminosity as compared with ordinary reflection from fairly non-selective surfaces. More generally the material is selective, and, as soon as it is in a state of aggregation in which ordinary diffuse reflection is predominant over scattering, color appears and the luminosity rapidly decreases. The change due to aggregation can be well seen in smoke or any of the usual experiments in scattering, and under suitable conditions the regions of blue, whitish, and heavily colored material can be separated.

Assuming that a mass of dust is dense enough to transmit no light directly, it will therefore have its highest coefficient of diffuse

[1] *Transactions Illuminating Engineering Society*, **9**, 593.

[2] *Philosophical Magazine*, **41**, 107, 274.

reflection when it is intrinsically fairly non-selective, coarse enough not to produce much scattering effect and presenting as nearly a smooth surface as possible, that there may be the minimum amount of multiple reflection. Hence no dust clouds or loosely aggregated surface can give the same reflective power as a smooth layer of the same material. Table I gives a list of such coefficients of reflection, mainly as determined by Nutting.[1]

TABLE I

COEFFICIENTS OF REFLECTION

Magnesium carbonate	89.4
Magnesium oxide, dry	88.1
Calcium carbonate, smooth	87.9
Calcium carbonate, brushed	86.1
Calcium carbonate, rough	83.4
Aluminum oxide	87.4
Borax	85.2
Snow, fresh, fine	84.7
Sodium chloride	81.9
Snow (Zöllner)	78.3
Cloud (Abbot)	65.0

The method used by Nutting leaves very little to be desired in point of precision, and one must regard the occasional figures given a few per cent higher as due to rough methods of testing. Most known terrestrial substances are noticeably colored and have even under the most favorable conditions much lower coefficients than these.

Therefore, while optical dust in clouds or layers may be more luminous than a rough block of the same material, there is not the slightest reason to suppose that its reflectivity can reach materially above 85 per cent, and, even so, only when the substance is solidly "white" and not in the form of any loose aggregation, cloud or otherwise.

Notably colored substances have reflectivities considerably lower than these mentioned and when massive, rather than in a smooth homogeneous layer, very much lower. Ordinary rocks, even when visibly white, do not come anywhere near the values just given. Table II shows the ordinary range which may be expected of them.

[1] *Transactions Illuminating Engineering Society*, 9, 593.

TABLE II

Substance	Albedo	Authority
Pumice	0.56	Wilsing and Scheiner
Yellow sandstone	.38	Wilsing and Scheiner
White quartzite	.25	Lowell
Whitish sandstone	.237	Zöllner
Clay shale	.16	Lowell
Marl	.156	Zöllner
Quartz porphyry	.108	Zöllner
Weathered sandstone	.10	J. Herschel
Gray syenite	.078	Zöllner

These, like the others, are specific reflectivities at small angles of incidence and consequently correspond to Lambert's albedo for diffusing material. In brief, the data on reflectivities show that no plausible terrestrial substance when in massive form can give anything like the reflectivity required to account for the observed brilliancy of the bright part of the *B* ring of Saturn. Assuming it to be a first-class diffuser, one therefore has to conclude that, whatever the structure of this annulus may be, the only terrestrial substances which can reasonably account for its albedo are of very light-colored matter finely subdivided so as to give the effect of an exceptionally white cloud. No mere collocation of satellites or similar small bodies, even if closely enough mustered entirely to prevent the transmission of light, can be counted upon to give the albedo approximating 0.80 required in this case. Nor indeed is such a collocation permissible, as we shall presently see, for dynamical reasons. In the case of the *A* ring, with the exception of its brightest portion, the reflectivity falls within possible limits, except for the reasons just stated. With the *C* ring the difficulty, as we shall later see, is not to account for a high albedo, but for one extraordinarily low considering even the well-known transparency of this structure.

The nature of the reflecting matter in the rings is beautifully disclosed by Wood's photographs in monochromatic light.[1] The yellow plate of his series shows the rings much as they appear visually, but in the violet and ultra-violet plates the *A* ring was very nearly as bright as *B*, and both notably brighter than any

[1] *Astrophysical Journal*, **43**, 310, 1916.

part of the ball, while in the ultra-violet plate the relative brightness of *A* is very conspicuous and Cassini's division is nearly obliterated. The *C* ring is almost unchanged, but shows a very slight brightening, which Wood suggested may be due to dust. In fact extremely fine optical dust is the only known material which is strongly selective for the violet and ultra-violet. The few substances which are fairly good reflectors for the ultra-violet are much better for the bright part of the spectrum, while from the λ^{-4} relation extremely fine dust would scatter more than seven times as much light in the 350 $\mu\mu$ to 300 $\mu\mu$ region as in the yellow. Considering the relative weakness of the incident ultra-violet radiation, selectivity of the order of magnitude necessary to account for the differences shown in Wood's photographs is not to be found in any massive substance. One therefore has to conclude that, whatever the general structure of the *A* ring from the visual standpoint, it is charged with the finest optical dust, which is also present about the *B* ring and permeates Cassini's division as well—visible evidence of the grinding process from collisions which makes up the history of the rings. The absence of any large quantity of the same dust in the *C* ring bears witness to its comparatively loose structure.

ISOPHOTAL LINES OF THE RINGS

The distribution of light in the rings is difficult to estimate visually, but photographically it is easy. The denser parts of a negative, however delicately graded, print white for a short exposure. More time drives through the detail preserved in the denser regions, while obliterating half-tones in the weaker regions in a common blackening. Hence if one starts with a well-timed and graded negative, prints of progressively lengthened exposure disclose the sequence of brightness in the various regions of the original very definitely, although it may be recognizable with difficulty in the primary image.

One can thus obtain for any photograph a set of isophotal lines representing the distribution of density in the original, a process which might be made of considerable service in the study of nebulae. Figs. 1, 2, 3, and 4 of Plate I are examples made from an enlargement of Barnard's Mount Wilson plate of November 19, 1911. The

PLATE I

FIG. 1

FIG. 2

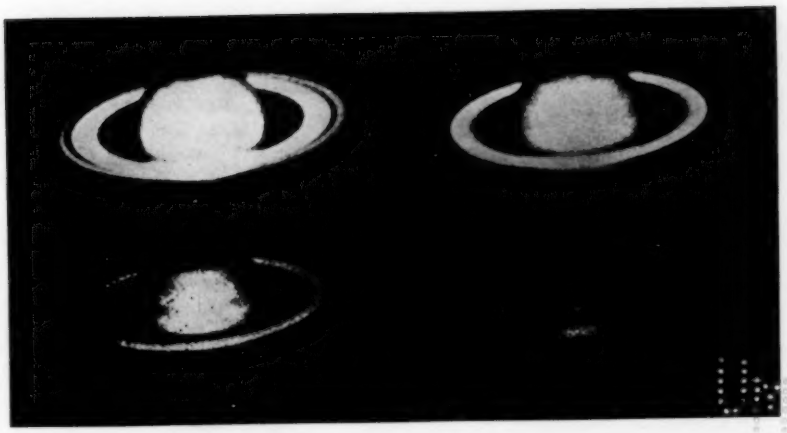


FIG. 3

FIG. 4

ALBEDO OF SATURN'S RINGS

writer could not conveniently lay his hand on a complete positive from this plate, but one from the same negative, containing part of the images, was available. This contained six images with major axis about 13.5 mm. The first thing notable is that in each image the ball is distinctly seen through the *A* ring, a fact pointed out by Hepburn[1] in measuring a positive from another of Barnard's photographs of November 19, 1911. In the plate examined by the writer the difference in contrast was small, only 2 or 3 per cent, but unmistakably showing that the albedo of the planet's ball at the point of crossing was sufficient to add something of brightness even in spite of the shadow of the ring. The same phenomenon is shown in some beautiful negatives of Saturn taken by Lowell in 1911, 1912, and 1916. Visually this small contrast seems to be spoiled by irradiation.

Fig. 1 shows *A* and *B* well, but the original exposure suppressed *C*. *A* is merely slightly narrower than usual. Fig. 2 shows *A* as a mere ghost, only its more brilliant annulus showing at all. Ring *B* in this figure is narrowed about 20 per cent. Fig. 3 shows *A* obliterated and *B* still further narrowed, and Fig. 4 discloses only a shred of *B*, the most brilliant part. In carrying out this printing process for isophotal lines the outer edge of *A* shrinks rapidly, accompanied by a widening of Cassini's division at the expense of both *A* and *B*, the latter rather the less, and a slow expansion of the inner diameter of *B*. Cassini's division in all photographs, in fact, is abnormally wide, about double the visual figure, showing the tenuity of the reflecting matter at its margins. It is as if the sweeping action of the satellites were somewhat slovenly, leaving careless margins and dust scattered in the path. Fig. 5 shows the situation to scale. The faint solid lines upon ring *A* mark the residual bright annulus. The thin solid line just within Cassini's division is the outer edge of the bright annulus of *B*. The broken line within is the inner edge of this residual, and the broken, and thin solid, lines near the inner rim of *B* show a late and early stage of the progressive disappearance of the light. These facts show plainly the cause of the discrepancy as between such measurements as Hepburn's, however carefully made, and the visual figures. The

[1] *Monthly Notices*, **74**, 721, 1914.

structure disclosed, then, is a very tenuous outer edge of A, gradually and then rapidly increased optical density up to a point just outside of Cassini's division. Within this is a faint portion, a similar faint portion on the extreme exterior of B followed by an annulus of great density fading out at first slowly and then more rapidly toward the inner edge of B. Within this lies the tenuous ring C fairly dense at its outer rim and fading into transparency at the inner.

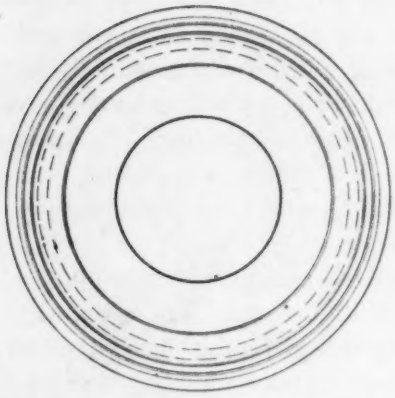


FIG. 5

The observations of Barnard,[1] Lowell,[2] and earlier of Bond[3] show that when the rings are seen nearly edge-on they become invisible save for certain condensations at two points, the outer having its center just inside Cassini's division substantially at the location of the residual annulus of Fig. 5 and extending outward just across Cassini's division. The inner has its maximum of light nearly midway of C and fades out in both directions. The several observers do not agree precisely, although substantially, as might be expected in viewing so faint and difficult objects. All agree in their nebulous appearance (Barnard describes it as pale gray) and apparent thickness when the main line of the rings was beyond vision. This thickness cannot be wholly charged to irradiation, which is a function of brightness certainly not conspicuous in mere nebulous ghosts of lines. One is therefore forced to the conclusion that whatever difference of density there may be there is also a heaping up of scattered dust which reflects the light. Such heaping also is indicated in some of Trouvelot's observations and by others, particularly Pratt,[4] in observing the shadow of the ball on the rings. The observations, like numberless others regarding striations on

[1] *Monthly Notices*, **68**, 346, 1908.
[2] *Lowell Observatory Bulletin*, No. 2.
[3] *Harvard Annals*, 1.
[4] *Monthly Notices*, **44**, 411, 1884.

the rings, and particularly concerning Encke's division, make it clear that the ring structure is far from invariability.

THICKNESS OF THE RINGS

Save for the condensations, the rings when fairly edge-on are beyond the reach of all but the very largest telescopes and of doubtful visibility in these. Since the matter of the rings is equally reflective at all angles, an edge-on view with a deeper mass of reflecting matter in the line of sight should be at least as bright as in any other aspect. Hence the disappearance of the rings indicates that the thickness is too small, at their effective albedo, to render them visible as linear bright objects.

The *minimum visible* for a bright line on a dark background is a quantity roughly ascertainable. The experiments of W. H. Pickering at Arequipa with markings on artificial disks[1] show that in general a line of eight or ten times its breadth can be recognized as easily as a round spot of the same total area. These were black markings on a white disk, by daylight, through 18 km of air. The experiments of Aubert[2] half a century ago showed that, as between black on white and white on black, the latter was the more easily visible, so that for a bright line on the background of a dark sky the visibility-ratio should be more favorable.

Next, one can turn to the dimensions of celestial objects actually visible. The smaller satellites furnish examples having albedo of the order to be ascribed to Saturn's ring structure. From photometric data W. H. Pickering[3] shows that Rhea does not exceed 0".1 in diameter, i.e., about 600 km, while Hyperion has a diameter of about one-fourth this, i.e., 0".025, 150 km. On this basis a line of similar brightness should be visible down to 15 km. Deimos and Phobos from the photometric data have diameters of the order of 10 km, Deimos probably less, approximately 0".02, indicating again visibility of a bright line at the distance of Mars not exceeding 1.0 km, and at the distance of Saturn not over 15 km. Other objects, e.g., the smaller visible asteroids, lead to similar results,

[1] *Harvard Annals*, **32**, 144.

[2] *Physiologie der Netzhaut.*

[3] *Harvard Annals*, **61**, 85.

but these examples are taken as not excessively difficult for fairly large telescopes.

Another line of attack is through the visibility of thin lines to the naked eye, having regard to the loss of visibility in telescopic vision. Lowell[1] records experiments on the visibility of a dark wire against the sky and finds that it was plainly perceptible down to an angular diameter of 1″ and with some difficulty down to 0″.7. A rusty iron wire against the sky does not present a very favorable contrast, and visibility increases with contrast very notably.[2] At the distance of Mars, Lowell reckoned that, taking into account the losses in telescopic vision, he could detect lines down to less than 1 km, which at Saturn's distance would be substantially 14 km, and in general this would be a safe estimate for a bright line against dark sky in good seeing. Earlier, Barnard[3] records the observation of a dark wire against the sky, subtending an angle of but 0″.44, which makes the case still stronger. The writer has obtained figures down to 0″.46 for white thread against black paper. From these data it seems highly probable that the substantial layer of Saturn's rings does not exceed 15 km in thickness. This value, 0″.0025, concurs closely with Russell's suggestion of 0″.003, made on a different basis (*Astrophysical Journal*, **27**, 233, 1908).

The condensations appear much thicker but a great deal less brilliant than a genuine edge-reflection at anything like the normal albedo of the rings. Barnard[4] records their apparent thickness as a few tenths of a second, perhaps up to 0″.5. As nearly all observations on them were made when the rings still presented a minute angle of obliquity the real thickness is doubtless materially less, but there is clear evidence of annular heaping up of matter differing from that in the central ring body. Light, from whatever source, cannot be reflected from a void, and since the brightness of the condensations is from Russell's[5] figures less than 1 per cent that of a satellite, the albedo of the condensation as a whole is extremely

[1] *Lowell Observatory Bulletin*, No. 2.

[2] Aubert, *loc. cit.*

[3] *Popular Astronomy*, **5**, 2, 1897.

[4] *Loc. cit.*

[5] *Loc. cit.*

small and the matter so scattered that it becomes visible only at extreme obliquity, like a layer of mist, invisible save when seen edgewise.

The inner condensations indicate such a mass of diffused matter surrounding the central plane of ring *C*, and this would account for the repeated observations, especially from its shadow, that ring *C* is thicker than the general plane, e.g., Wray.[1] These facts lead to a cross-section of the ring system approximating Fig. 6, including all the space occupied by visible matter irrespective of its character, on the scale of $1'' = 10$ mm horizontally and somewhat exaggerated vertically.

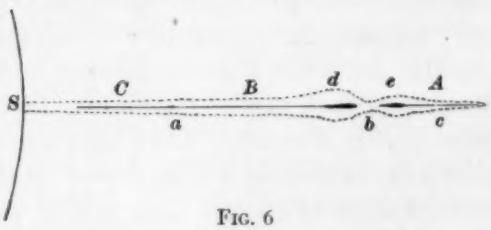


FIG. 6

The situation presented is that of a very thin, nearly plane ring of relatively dense matter with a permanent discontinuity at Cassini's division (*b*) and possibly temporary ones at *a* and *c*, Encke's division, and varying in density rather than thickness, save perhaps at the points *a*, *d*, and *e*, where increased density is likely to lead to more frequent collisions and departures from the average ring plane. Surrounding this much thicker stratum of extremely tenuous matter there are fine dust and detritus from collisions in the ring proper, somewhat more dense and extensive at *d* and *e*, where the grinding chiefly goes on, greatly and more uniformly scattered about ring *C*, where collisions are rare and particles can pass more freely out of the ring plane. Note that the thicker portions reflect visibly only at very oblique incidence.

SOURCE OF LIGHT IN CONDENSATIONS

With rings nearly edge-on the outlying matter of *C* receives sunlight directly in one aspect and by penetration of the tenuous ring in the other. In either case there is back reflection which, as will be presently seen, may be of very perceptible amount. Such is

[1] *Monthly Notices*, **23**, 86.

the explanation offered by Barnard[1] and concurred in by W. H. Wright.[2] Bond postulated reflection from the inner edge of *C*, a condition demanding far too great density of matter there. Russell[3] shows in addition that the light reflected to the ring by the ball of Saturn is sufficient to be a perceptible quantity and doubtless this co-operates with the direct sunlight reflected in the ring haze. The sources must evidently coact and their summation is what we see. Barnard did not catch the condensation when the earth was slightly on the sunlit side of the ring, and hence laid aside the idea of a thickened mass. He shows, however, the condensations extending on both sides of the faint edge line of the ring, and they seem to have been so seen by Lowell[4] both as luminous objects and in the shadow on the ball. But neither observer could see any trace of them from the sunlit side.

Bond[5] in 1848 saw the same objects only from the shadowed side of the ring, but changing sides with reference to the central plane of the rings as the earth passed from one aspect to the other, a dissymmetry shown in some of his drawings. Wray,[6] on the other hand, saw a nebulous band of light about the *C* ring on both sides of the line of the edge reflection. His description leaves small chance for any effect of irradiation as the cause, which indeed is altogether unlikely in so faint an object. The outer condensation escaped him, though both sets were contemporaneously seen by Otto Struve.[7]

The weight of the evidence points to outlying regions of very much scattered matter lying chiefly about the *C* ring and on each side of Cassini's division. The latter piles furnish a simple explanation of the outer condensations. Cassini's division is not entirely clear of matter, as many observers have noted, e.g., Secchi; the adjacent piles are anything but dense at the edge, and there is ample opportunity for the sunlight not only to be reflected from the masses of haze but to stream through Cassini's division and light

[1] *Loc. cit.*

[2] *Astrophysical Journal*, **27**, 363, 1908.

[3] *Ibid.*, 230, 1908.

[4] *Loc. cit.*

[5] *Harvard Annals*, **1**.

[6] *Monthly Notices*, **23**, 85.

[7] *Ibid.*, **32**, 86.

them up all down the line. The situation seems to combine the effect of deep reflection of the penetrating light to the shadowed side, with inner edge reflection, not as Bond pictured it, but again as deep reflection from a stratum of mist seen edgewise. The double inner condensations seen by Aitken[1] hint at lessened density at the point *a*, such as was earlier noted by Secchi. That the condensations disappeared when the earth was exactly in the plane of the rings[2] is to be expected in view of the great extent of feebly reflecting optical dust, some 15,000 km wide, drifting about ring *A*, interposed between the observer and any point at which perceptible light could stream through. The faint condensations could be seen only in a fairly unobstructed line of sight. Then, as Barnard[3] notes, the great obliquity gives by perspective enhanced apparent brightness.

This implies deep reflection from a tenuous cloud of highly diffusing matter like optical dust, for neither a compact diffusing surface nor an aggregation of stony masses for which in general Lambert's law does not hold could give the increased brightness observed by Barnard and others. The larger the proportion of dust to massive matter the more striking this increase; hence the relative brilliancy of rings *A* and *C* in very oblique view.

No mere meteoric constitution of the rings can sufficiently account either for the observed albedo of their brighter portions or for the complete invisibility of the condensations from the sunlit side of the structure.

LIGHT-TRANSMISSION OF THE RINGS

Ring *C* was found by Bond to be transparent enough to show the limb of the planet through its whole extent, and, while both *A* and *B* were long considered wholly opaque, *C* is evidently only very slightly absorbing, else with any probable material it would show much higher albedo than it does. The only quantitative data on the transparency of ring *C* rest on Barnard's photometric estimates of the eclipse of Japetus by the rings, made November 1, 1889.[4]

[1] *Lick Observatory Bulletin*, No. 127.

[2] *Monthly Notices*, **69**, 623, 1909.

[3] *Loc. cit.*

[4] *Astronomy and Astrophysics*, **2**, 120, 1893.

Visual estimates were made by comparison with two other satellites in the same field, so that the conditions were fairly good for recording the change in magnitude as Japetus emerged from behind the shadow of the ball, entered the shadows of the crape ring, and then disappeared in the *B* ring. Barnard's estimate of the mean loss of light for the body of the crape ring was $0^m.412$. This means that the net transmission coefficient was 0.68 and the extinction 0.32. But the obliquity of the ring plane in this case is about 11°; hence, considering the component bodies to be distributed uniformly and roughly spherical, the total light transmitted at normal incidence would be approximately 0.94 for ring *C*. The data on the photographic transparency of ring *A* do not permit estimating it even roughly, merely showing that the part of the ball covered by the ring has albedo high enough to return enough light through the ring to give a small contrast with the light directly reflected by the rings.

There is, however, a recent observation by Ainslie[1] which gives a clue to the visual transparency of ring *A*. Ainslie observed on February 9, 1917, an occultation of the star B.D.+21°1714, of about magnitude 7 by Saturn's ring, also noted by Knight (*loc. cit.*). It was seen just grazing ring *B* in Cassini's division, along which it passed very obliquely with small diminution of brightness, and then passed behind ring *A*, through which it remained visible, but dimmed, until its emergence. Ainslie estimated that the star lost 0.75 of its brightness in its occultation, but it twice showed brightening, not to its full magnitude, once in passing the approximate place of Encke's division and again slightly farther out, the changes in brightness being rapid but not instantaneous. Clearly Encke's division, like Cassini's, is not abrupt and contains reflecting material. Taking the transmission of the ring, then, at about 0.25, the transmission at normal incidence should be from the ring angle of about 26° somewhere about 0.6, a figure fairly consistent with the albedo of a mass containing considerable optical dust.

As to ring *B*, there is no direct evidence of transparency, although no occultation of a fairly bright star has yet been observed. In view of the faintness of the inner part of *B* sometimes observed,

[1] *Monthly Notices*, **77**, 456, 1917.

e.g., Secchi,[1] it may well be permeable to light. Perhaps at such times as shown in Secchi's curve of estimated brightness a division between *B* and *C* appears (Fig. 7), together with enhanced brightness of *C*.

STRUCTURE OF THE RINGS

The data cited merely show the general shape of the rings, a thin, relatively plane annulus tenuous in the *C* region and overlaid there with a mist of scattered dust, a denser structure nearly opaque at considerable angles of incidence in the *B* region and thickened heavily near its periphery by optical dust, a stable, somewhat clouded division beyond, and then the *A* ring thickened near its inner edge and fading off to nothing outside, overlaid by the optical dust conspicuous in its photographic albedo. Besides the Cassini division there are the somewhat unstable Encke's division in the *A* ring and now and then impermanent traces of others, especially in *A* and at the inner limit of *B*. On this last score Fig. 7 is conclusive. As to the general constitution of the rings, Cassini's guess, Maxwell's analysis, and Keeler's physical proof unite upon a loose structure of discrete particles, whether of satellites or sand. Telescopic evidence merely states that there are no satellites separately visible. The faint sparkling noted by Trouvelot[2] is a familiar phenomenon in physiological optics which on divers occasions deceived Sir W. Herschel in studying nebulae.

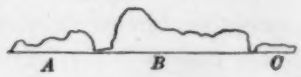


FIG. 7

Nor does Roche's famous paper,[3] more quoted than read, give any substantial help in solving the problem. Roche was dealing with the case of a fluid mass held together by its own gravitational forces, and determined the limits of its stability. In the case of Saturn he showed that the limiting distance from the planet's center was, for equal density of the two bodies, 2.44 times the radius of Saturn—a point slightly outside of the ring system. For unequal densities $r' = 2.44 \sqrt[3]{\frac{d}{d'}}$, r' being the new critical radius

[1] *Monthly Notices*, **16**, 50, 1856.

[2] *American Journal of Science* (3), **11**, 447.

[3] *Memoirs of Montpellier Academy*, **1**, 243.

and δ and δ' the densities of planet and satellite. Taking δ' at the average density of such small celestial bodies as we know, 3.6, $r'=1.42$, reckoning the density of Saturn at 0.7. This point lies just within the inner edge of ring *B*. Perhaps the only conclusion to be drawn is that inside this limit there can be no large stable aggregation of particles maintained by gravitational attraction alone, but it is interesting that this limit falls just at the point which divides the tenuous from the dense. At least as bodies drew in toward the critical radius they would be subjected to gravitational disruptive forces which would facilitate their breaking up under collisions.

The relations between the light reflected and transmitted by a body of discrete particles furnish, however, some clues to the fineness of the structure concerned.

Conceive the whole mass, assumed uniform, to be divided into laminae containing on the average one layer of reflecting bodies, i.e., of thickness equal to the average separation of the bodies in space. Let T_0 be the coefficient of light-transmission by such a lamina, S_0 the space-factor of the lamina, i.e., the proportion of the area intercepted by the solid matter, and k the apparent albedo. Then for light incident at any angle for which S_0 is found or computed, the intensity I' of the light, reflected backward, neglecting terms in higher powers of k, is:

$$I'=IS_0k(1+T_0^2+T_0^4+T_0^6 \ldots\ldots T_0^{2n-2})$$

while the total transmission coefficient of the structure is $T_n=T_0^n$. The foregoing series must be regularly summed term by term if the terms be few or converge rapidly. For a very large number of slowly converging terms the sum to a first approximation is, since $S_0=1-T_0$, and the series is a familiar one with the odd powers absent,

$$I'=I(1-T_n^2)\frac{k}{2}.$$

Unluckily the difference in I' as computed for few and many laminae is too slight to give a trustworthy value of n from photometric data alone, thus blocking an otherwise promising line of attack on the problem of structure.

Barnard's observations give for ring *C*, at 11° 10′ inclination, $T_n=0.68$, and assuming k at 0.2, the average for small celestial bodies as we know them, we find for a loose structure of many laminae $I'=0.054$ as the apparent albedo of the swarm seen from the given angle. This is little in excess of that from black paper, and is perhaps no higher than is required to make the ring visible against a sky far from devoid of light, and in the form of a narrow strip between two rather brilliant surfaces. Obviously the scattered particles about the ring, which give edgewise the faint shimmer of the inner condensation, cannot, if Russell's estimate of its intensity is near the truth, add perceptibly to the light of ring *C* save at very large angles of incidence.

Considering now the light transmitted through the mass and reflected downward instead of backward at a similar angle, this component, neglecting terms in k^2 and higher powers, is

$$I''=S_0kT_n.$$

For the case of many slowly converging terms this approaches I'. There may therefore be illumination of the shadowed side of ring *C*, by percolation of light, something approaching the brightness of the sunlit side at the same angle, but percolation cannot account for an increase of physical brightness, save as k increases, as it does for fine particles.

In similar fashion, taking Ainslie's rough estimate[1] of the light absorbed in ring *A*, we have

$$T_n=0.25,$$

angle of ring 26°. At $k=0.2$, as before, the apparent albedo of the swarm would be 0.0937, which is much lower than other observations indicate. To account for an albedo of 0.3, which seems approximately right, the actual reflectivity of the matter should be about

$$0.2\frac{0.3}{0.0937}=0.64,$$

a value much too high for any plausible massive solid but a perfectly good value for dust, of the existence of which in ring *A* the photographic evidence leaves no doubt. In this instance the

[1] *Loc. cit.*

brightness of the occulted star was likely to be underestimated from its bright background, leading to a larger value of T_n, which could hardly exceed 0.5, however, without bringing the derived albedo too high for dust known to scatter light selectively and hence lacking conspicuously in brightness.

Concerning ring *B*, it is so far as known opaque at the available angles of incidence, and hence of greater volume-density, if of material similar to rings *A* and *C*. That such is the case follows from the necessary exchange of matter due to changes of velocity, both + and −, from the inevitable collisions due to the gravitational forces. Maxwell[1] has shown the complicated compressional wave systems that arise in the plane of the rings.

Maxwell[2] also shows that the condition of stability in a ring of discrete bodies is that the volume-density shall not exceed 1/300 the density of the planet. This implies that at the very utmost ring *B* has a volume-density not exceeding 0.00233.

One cannot suppose that the actual density leaves the ring on the verge of complete instability, and must reckon on a moderate factor of safety. Taking this at 2, assuredly small enough, the volume-density of this *B* ring cannot exceed 0.00116. That is, for each cubic meter of volume there cannot be more than 1160 cc of matter of unit density, and bulk for bulk the ring space is lighter than air, as Maxwell pointed out. At the mean density of small bodies, 3.6, this volume of matter shrinks to 323 cc, equivalent to a sphere 8.5 cm in diameter for each cubic meter. The projected area of this is 57 sq. cm, and a structure thus proportioned, on whatever scale, would show at normal incidence $T_0 = 0.9943$. Ring *B* is substantially opaque so that one may reckon for normal incidence $T_n < 0.10$. Since $T_n = T_0^n$, one can make at least a rough estimate of n and hence of the scale of the structure supposed uniform. Thus, n is in the neighborhood of 400, i.e., a distribution of the volume-density indicated should be spread in at least 400 laminae to account for the apparent opacity of the ensemble. At the total thickness of ring already found, 15 km, the unit body would have a diameter of approximately 3 m. This figure would fall for any increase of the factor of safety in density.

[1] *Scientific Papers*, 1, 328 ff. [2] *Ibid.*, p. 338.

The fact that there is much optical dust around the brighter part of ring *B* makes the light transmission there chargeable to the denser structure very uncertain. A better point of attack is the outer portion of ring *C*, where Barnard's figures give a fair approximation to T_n, and where the structure, except in density, may be taken as comparable with the nearby portions of ring *B*, which are relatively free of optical dust and into which, except at occasional periods, ring *C* merges. Taking for example the point where the apparent transmission was 0.50, the real normal transmission amounts to $T_n=0.903$. The density here is evidently very much less than in the neighboring opaque edge of the *B* ring. Taking the density-ratio as 10 in round numbers, the material volume per m^3 is 32.3 cm^3, the corresponding sphere a little below 4 cm, the projected area 12 cm^2, and *n circa* 90. This leads to a unit body slightly less than 8 m in diameter.

Similarly, taking Ainslie's rough estimate of the transmission of ring *A* at 26° inclination, $T_n=0.57$; taking the volume-density at one-third that of ring *B*, the mass diameter per m^3 is 5.9 cm diameter, its projected area 27.3 m^2, *n circa* 200, and the unit mass of the ring about 4.3 m.

The figures depend obviously on the assumed values of the factor of safety of stability, but it seems clear that, taking Maxwell's limit as the basis, and reckoning the unit masses as for a homogeneous medium from the approximate light-transmissions, the diameters are of the same order of magnitude in the several rings and do not as a whole exceed a few meters. Indeed, unless ring *B* is precariously near to instability, considering its permanence, the unit masses are likely to be considerably smaller.

The point of the matter is that the observed albedos and coefficients of light-transmission are not consistent with bodies even approximating satellite dimensions. They indicate rather bodies of meteorite dimensions ranging from a few meters in diameter down to brickbats, chips, and dust, the last named being the most plentiful in the bright zones of rings *A* and *B*, where it appears to have its continuous origin in an increased volume-density of the larger masses. Very fine dust cannot remain stable under the pressure of radiation.

TOTAL MASS OF THE RINGS

Maxwell's limit at once assigns a superior limit to the mass of the rings. Leaving the *C* ring out of the reckoning and figuring the rest as uniform, at 15 km in thickness the total mass would be about 1/400,000. Taking into account the factor of safety in stability, and counting the density of ring *A* as above, with due allowance for *C* and noting that *A* fades away to the edge, it appears that the total mass of the rings can hardly exceed 1/1,000,000, and is likely to be much less.

THE RÔLE OF LIGHT-PRESSURE

If we bear in mind the considerable amount of drifting dust, which increases the albedo of Saturn's rings beyond any reasonable figure for massive matter, it is evident that the pressure of radiation may play a very perceptible part in the distribution of the dust. At the density here taken for the material of Saturn's rings and since solar gravitational force is balanced by its pressure of radiation for diameters not greatly in excess of a wave-length, for the much finer matter which takes part in the scattering of violet and ultra-violet light the repulsive force of light-pressure is relatively very much greater, and becomes a perceptible quantity even with reference to the gravitational attraction of Saturn.

With respect to the gravitational effect of Mimas, the perturbative action of which is chiefly responsible for Cassini's division, light-pressure is a very large quantity. It is clear, therefore, that Cassini's division cannot be swept clear of dust, as against the effect of the pressure of radiation in pushing it back and forth across that division in the nearer and farther parts of the ring system. To go farther, the same force tends to push down into the ring plane dust which through collision is projected out of it. Since the cosmic grinding-mill is near the outer edge of ring *B*, most dust will be generated here, and the effect of light-pressure will be to push it back in the nearer part of the orbit and out into ring *A* in the farther. Matter thus transferred will on the whole tend to be driven down into *A* rather than back again into *B*, owing to the obliquity of the rings, and the final result of this constantly though slowly acting force should be to transfer much fine optical

dust from its zone of origin outward, which furnishes one good reason for the high albedo of ring *A* for ultra-violet light. With respect to ring *C*, a certain small amount of matter must inevitably be transferred to it by light-pressure, but in the main would be driven into *B* again. The slight added brilliancy of *C* found in Wood's photographs bears evidence of the existence of such matter there, and a like origin may reasonably be sought for the considerable volume of extremely tenuous matter which seems to be the source of light in the inner condensations.

The outstanding difficulty regarding the condensations is that they have been visible only on the shadowed side of the rings, never from the sunlit side. The effect of light-pressure, however, is to keep the mobile dust-cloud always toward the shadowed side, and hence hidden by the shadow of the main ring when the inclination of the rings is very small. The same force acts to depress fine dust upon the illuminated ring surface. Obviously the dust is most free to scatter over the surface when the rings are nearly edge-on, which may account for the exterior hazy ring observed by Fournier[1] and Schaer.[2]

The relative masses and distances of Saturn and sun from the region of ring *C* indicate that the acceleration due to radiation-pressure for particles, say from μ down, is of the order of 0.1 mm per second, which amounts to nearly 350 km in a day. Hence as the sun rises above the plane of the rings there is a downward component gradually displacing the fine dust into and below the ring plane. Here it would become visible only as the angle of view permitted enough light to filter through the body of the ring to illuminate the matter below. It is unseen from above simply because it is not there. The dust mass would thus oscillate between two positions of stability determined by the attraction of Saturn, though the finer particles with relatively high acceleration would naturally find their way to the planet. There is thus strong circumstantial evidence, throughout the rings, of radiation-pressure acting freely on a very extended mist of fine particles, in ring *C* several hundred km thick and 16,000 km wide—a demonstration of Maxwell's theorem on a truly colossal scale.

[2] *Scientific Papers*, **176**, 64, 1908. [1] *Astronomische Nachrichten*, **176**, 239, 1908.

The ensemble of the ring system thus presented is a substantially plane, very thin stratum of bodies of meteoric size swept permanently clear at Cassini's division, thinned and grooved elsewhere by the perturbations of the satellites, denser near the grinding zones of action, and thinning out away from them. Permeating and overlaying this is a loose body of widely scattered dust more and more tenuous away from the ring plane, and everywhere away from it so thin a cloud that it reflects no visible light save when seen in great thicknesses edgewise. This floating spin-drift of the ether, thin as autumn haze, is billowed and wind-driven by the gravitational waves in the ring planes, and by the ever-acting pressure of radiation which sweeps the particles back and forth over the rings, and drives them down upon and through the sunlit side to be visible below when and where the light can filter through to illuminate them. It is a scene of perpetual change steadied only by the mass of the whirling ring plane itself.

BOSTON, MASS.
February 4, 1919

THE STARK EFFECT FOR METALS[1]

By T. TAKAMINE

In 1917 J. A. Anderson[2] developed a method suitable for examining the Stark effect for metals having a relatively high melting-point. The apparatus and methods used in the present investigation are essentially the same as employed by him, excepting a slight modification in the construction of the cathode and the grating spectrograph.

APPARATUS AND METHODS

In his experiments Anderson used a cathode having a diameter of about 13 mm. In order to attain a much stronger electric field, the diameter of the cathode, in the present work, was decreased to 2 to 5 mm.

In Fig. 1, *A*, a cylinder of the metal which is to be studied, is put into the iron frame *C*. *B* is a disk of fused silica having a hole in the center. The upper end of the cathode *A* extends about 0.3 mm above the upper surface of *B*. *D* and *E* are silica tubes ground to fit tightly together. *D* has a vertical slot, about 1 mm in width, which extends down to the level of the upper surface of *B*. The upper end of the slot was covered by a small piece of metal. *F* is the platinum wire leading down to the electrode below.

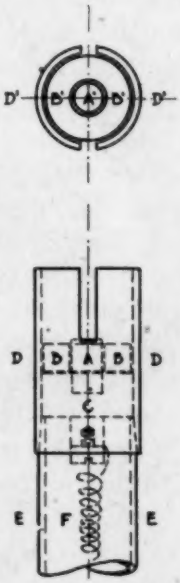


FIG. 1

The grating spectrograph is essentially the same as described by Anderson, excepting that a telescope objective of 5-foot focus, made by Brashear, was used as the collimator lens, and a Bausch & Lomb Tessar of F/4.5 aperture and 40-cm focus as the camera lens. A Voigtländer Heliar lens of 15-cm focus and a double-image prism were placed between the source and the slit of the

[1] *Contributions from the Mount Wilson Observatory*, No. 169.

[2] *Mt. Wilson Contr.* No. 134; *Astrophysical Journal*, **46**, 104, 1917.

spectrograph, so that two images of the uncovered portion of the slot appear one above the other on the slit. The bell jar containing the cathode and the anode is pumped out until the Crookes dark space has a length of 1 or 2 mm. Direct current to produce the discharge is supplied by a set of high-potential generators, each giving about 800 volts, connected in series. The number of dynamos used varied from 16 to 24. A milliammeter and a water-resistance were connected in series with the discharge tube, the current being in general between 20 and 40 milliamperes. The exposures varied considerably according to the cathode metal, ranging from a few minutes up to 2 hours. In order to avoid excessive heating of the cathode, the discharge was passed intermittently, the bell jar being kept cool by means of an electric fan.

The intensity of the electric field was determined by using Stark's data for the "Grobzerlegung" of the Balmer lines Hβ, Hγ, and Hδ;[1] its maximum value at the cathode ($E_{max.}$) varied from 28,000 volt/cm to 75,000 volt/cm.

With the cathode of smaller diameter used in the present work the distribution of the electric field in the Crookes dark space was found to be parabolic, as noted by Yoshida and the author.[2]

It has already been noticed by Anderson that the spectra of metals used as the cathode are excited to greater brightness when the residual gas is either air or oxygen than when it is hydrogen. In the present work air proved to be preferable to either oxygen or hydrogen. It would seem that the presence of nitrogen acts favorably in producing bright metallic spectra.

RESULTS

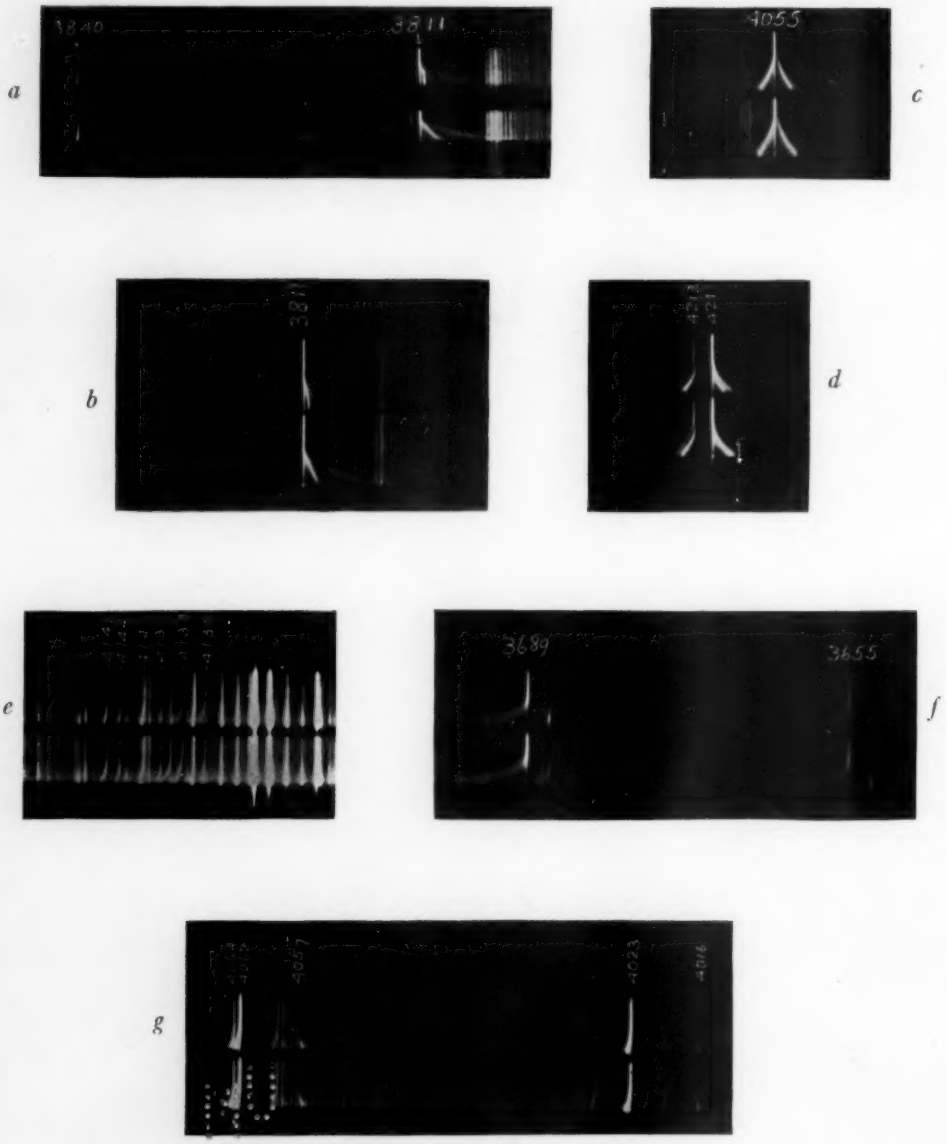
The spectra of the following ten metals were examined: Ag, Au, Co, Cu, Fe, Mg, Mn, Mo, Ni, W.

Of these, no affected lines were observed in the spectra of Mn and W. In the course of the experiments a few lines of Na, N, and O were also found to be affected by an electric field. The spectra of Co, Fe, Mn, Mo, and Ni were fairly rich in lines, while the rest of them showed a rather small number of lines. The color of the

[1] *Elektrische Spektralanalyse chemischer Atome*, 1914.

[2] *Memoirs of the College of Science* (Kyoto Imperial University), **2**, 137, 1917.

PLATE II



TAKAMINE—THE STARK EFFECT FOR METALS

In each illustration the *s*-components are above, the *p*-components below.

a Silver $E_{max.} = 48{,}000$ volt/cm
b Silver $E_{max.} = 31{,}300$ "
c Silver $E_{max.} = 26{,}700$ "
d Silver $E_{max.} = 26{,}700$ "
e Cobalt $E_{max.} = 33{,}300$ "
f Copper $E_{max.} = 75{,}200$ "
g Copper $E_{max.} = 33{,}000$ "

glow in the Crookes dark space was a very bright green in the case of Ag, Cu, and Mg; fairly bright for Fe and Ni; and yellowish for Au, Co, Na, and Mo.

In the tables the first column contains the wave-lengths, the second the series numbers whenever known; the third and fourth give the displacements of the *p*- and *n*-components, respectively, with relative intensities on a scale of 1–10. The displacements were measured at the maximum electric field, $E_{max.}$, which is given in the fourth column. For some lines special remarks are given below the table.

Silver.—With silver as the cathode the brilliancy of the greenish glow in the Cookes dark space was extraordinary, and strong lines came out with a few seconds' exposure. The Stark effect for silver lines is remarkably similar to that for helium lines. Excepting the green lines, the rest of the first subordinate-series lines are spread out very widely on both sides, while those belonging to the second subordinate series are displaced in one direction only. Moreover, the former show a number of detached components just as in the case of helium. This is illustrated for the line λ 3810.85, in Plate II*a, b*. The only difference we notice between helium and silver is that in the case of helium all the detached components appear invariably on the violet side, while in silver they usually appear on the red side.

With respect to these detached components, it is important to note that Merton[1] has found that those belonging to helium can be arranged in a combination series. It seems very likely that the detached components of silver may have similar properties.

These detached components are not of sufficient intensity to appear in the negative glow if, indeed, they exist there at all; they appear with sufficient intensity for observation only in the electric field.

It is important to notice that there is another class of lines, first observed by Koch[2] in the helium spectrum, which shows this feature still more markedly. Their approximate wave-lengths are 4519, 4046, and 3930 A, and they begin to appear with an

[1] *Proc. Royal Soc.*, **95A**, 30, 1918.

[2] *Annalen der Physik*, **48**, 98, 1915.

TABLE I

SILVER

λ	SERIES	F p-COMPONENT		n-COMPONENT		$E_{max.}$ IN VOLT/CM
		Δλ	Int.	Δλ	Int.	
3682.45	IN, II6	v_3 − 5.00	2	.....	.....	48,000
		v_2 − 1.59	6	v_2 − 1.84	1	
		v_1 − 0.69	1	v_1 − 0.84	5	
		0	4	0	2	
		r_1 + 4.35	2	r_1 + 4.90	1	
		r_2 + 13.00	1	.....	.....	
3810.85	IN, I6	v_3 − 7.15	6	v_3 − 4.78	3	48,000
		v_2 − 1.77	10	v_2 − 1.40	3	
		v_1 − 0.41	2	v_1 − 0.43	10	
		0	5	0	5	
		r_1 + 5.21	3	r_1 + 5.36	4	
		r_2 + 11.04	3	.....	.....	
3840.74	IIN, II5	+ 0.15	2	+ 0.08	2	48,000
3981.87	IIN, I5	+ 0.13	3	+ 0.14	3	48,000
4055.44	.....	.....	.....	v_2 − 4.49	2	56,000
		v_1 − 5.20	8	v_1 − 3.21	6	
		0	4	0	3	
		r_1 + 3.33	10	r_1 + 3.29	10	
4081.7	.....	+ 0.88	3	+ 0.86	3	55,000
4206.8	.....	+ 1.46	2	+ 1.28	2	55,000
4210.87	IN, II5	.....	.....	v_2 − 3.54	3	56,000
		v_1 − 3.80	10	v_1 − 1.91	10	
		0	6	0	6	
		r_1 + 4.23	5	r_1 + 3.99	5	
4212.76	IN, I5	v − 3.47	3	− 2.98	3	56,000
		0	5	0	5	
		r + 2.53	3	+ 2.70	3	
4226.55	.....	+ 2.33	3	+ 2.14	3	56,000
4476.31	.....	+ 0.04	10	+ 0.02	10	48,000
4668.58	.....	+ 0.03	10	+ 0.03	10	48,000

REMARKS

λ 3810.85 The behavior of v_2 in the p-component and v_1 in the n-component are anomalous. A companion line at +0.86 A has a violet component.

4055.44 In both the p- and n-components v_1 is detached from the main line by 0.2 A.

4081.7 The line appears only in a strong electric field.

4226.55 The line appears only in a strong electric field.

electric field considerably stronger than is required in the case of the detached components discussed above, and gain in intensity very rapidly as the field-strength is increased. According to Koch these new types of lines, which, for convenience, we may call lines of Koch's type, can be arranged in a combination series. The silver lines at λλ 4081.7 and 4206.8 are evidently of Koch's type.

The line λ 3682.45, which forms a doublet with λ 3810.85, is quite similar to the latter in its mode of decomposition. In both the *p*- and *n*-components of these two lines the violet components v_1 and v_2 show an anomalous behavior. In a weak field they are displaced approximately in proportion to the field-strength; as the field increases this law ceases to hold, and in a strong field the displacement becomes practically constant. As will be seen in Plate II*b*, the phenomenon is especially marked for v_1 of the *n*-component. The intensity of these *p*- and *n*-components is, so to speak, complementary. In fact v_1 is weak and v_2 strong in the *p*-component, while in the *n*-component exactly the opposite holds. As shown in Plate II*a*, *b*, the line λ 3810.85 is accompanied by a weaker line λ 3811.7, which has a violet component only.

The electric effect for the two neighboring lines λλ 4210.87 and 4212.76 is shown in Plate II*d*. These two lines, together with the line λ 4055.44 shown in Plate II*c*, form a doublet of the first subordinate series. The line λ 4212.76 behaves quite like λ 4055.44, while the line λ 4210.87 has its red component detached and at a considerable distance.

Gold.—As shown in Table II, only three affected lines of this element were observed.

TABLE II

GOLD

λ	*p*-COMPONENT		*n*-COMPONENT		$E_{max.}$ IN VOLT/CM
	Δλ	Int.	Δλ	Int.	
3796.15.....	−1.61	6	$\begin{cases} v_2 - 1.57 \\ v_1 - 0.52 \end{cases}$	$\begin{matrix} 3 \\ 5 \end{matrix}$	21,000
4084.31.....	+0.05	7	+0.04	7	30,000
4128.80.....	+0.32	4	+0.25	4	30,000

TABLE III

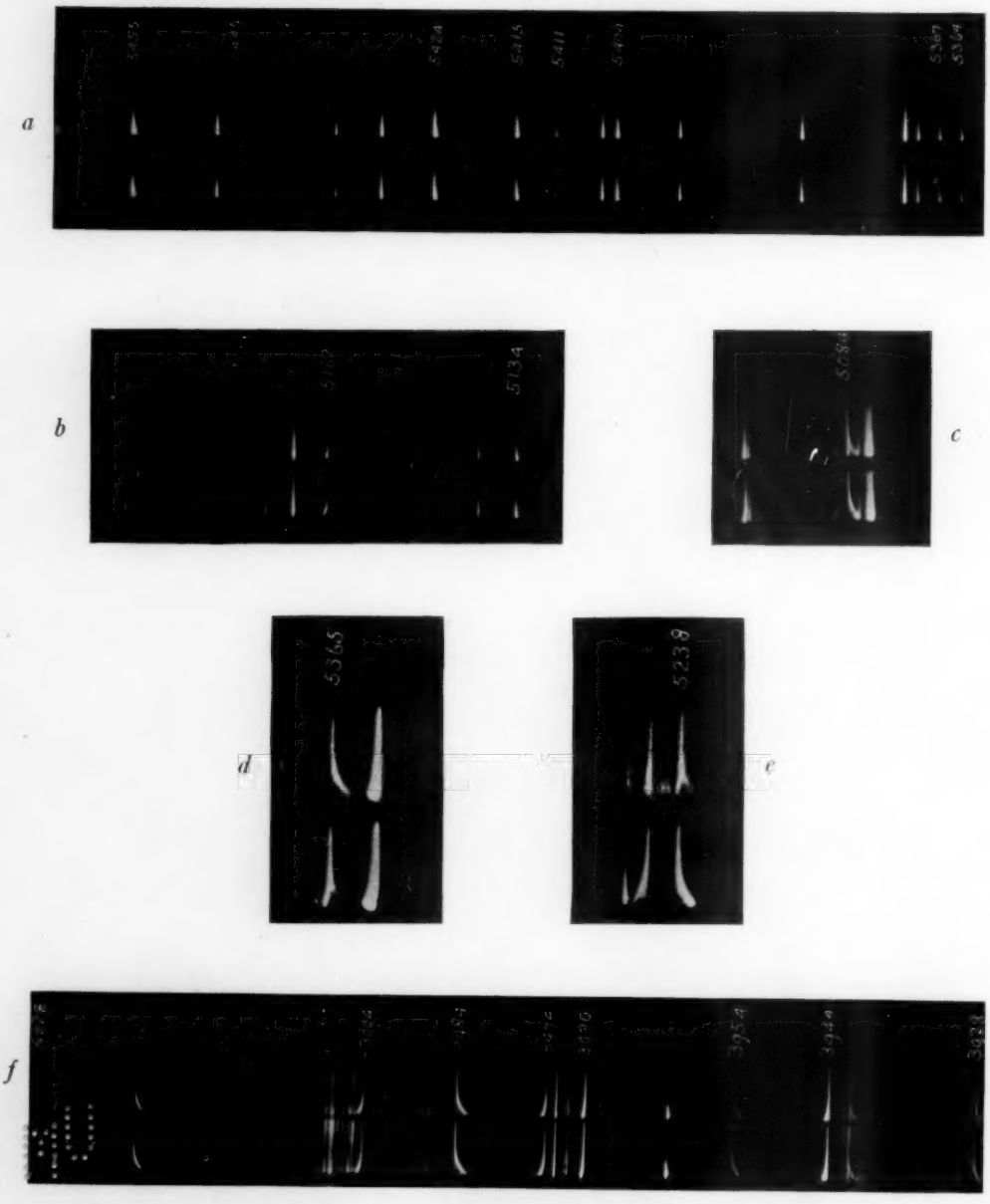
Cobalt

λ	*p*-Component		*n*-Component		$E_{max.}$ in Volt/cm
	Δλ	Int.	Δλ	Int.	
3664.7	+0.11	1	.....	.....	33,300
3702.4	+0.04	5	.....	.....	33,300
3704.17	+0.05	4	.....	.....	33,300
3732.52	+0.05	6	+0.05	6	33,300
4045.54	+0.11	6	+0.20	5	33,300
4072.43	+0.44	5	+0.35	4	33,300
4096.11	+0.71	5	+0.47	4	33,300
4126.1	−0.84	2	−0.74	1	33,300
4128.3	+1.11	1	.....	.....	33,300
4129.86	−1.75	3	−1.76	2	33,300
	+1.18	2	+0.98	1	33,300
4130.62	+1.36	4	+1.11	4	33,300
4132.41	+0.20	7	+0.20	7	33,300
4133.94	+1.20	4	+0.96	4	33,300
4135.85	+1.65	5	+1.33	2	33,300
4137.52	+0.52	3	+0.41	2	33,300
4139.60	+0.64	5	+0.49	5	33,300
4140.50	+0.70	4	+0.71	3	33,300
4141.35	+1.08	1	.....	.....	33,300
4143.98	+0.63	5	+0.52	4	33,300
4144.52	+0.88	5	+0.65	4	33,300
4158.59	+2.06	2	.....	.....	33,300
4164.7	+0.87	2	.....	.....	33,300
4177.77	+0.53	3	+0.41	2	33,300
4198.62	−1.71	3	+1.10	2	33,300
4200.0	+1.27	2	+1.00	1	33,300
4201.6	+1.53	4	+1.37	3	33,300
4212.2	+0.78	3	+0.78	2	33,300
4215.70	+0.15	1	+0.08	1	33,300
4220.59	+0.12	2	+0.26	2	33,300
4221.21	−0.06	2	−0.05	2	33,300
4223.93	+1.25	2	+1.25	2	33,300
4307.57	−0.53	3	−0.40	2	33,300
4459.8	−0.40	4	−0.27	3	33,300
4489.1	+0.46	4	+0.60	3	33,300
4512.8	+0.70	2	+0.82	2	33,300
4514.3	−0.06	4	−0.05	4	33,300
4517.24	+0.65	2	+0.10	2	33,300
4550.8	−0.77	2	−1.02	2	33,300
4557.2	+1.23	2	+1.25	2	33,300
4591.8	−0.08	8	−0.10	8	33,300
4660.2	+0.42	2	+0.44	2	33,300
4704.57	−0.24	2	−0.25	2	33,300
5336.36	−0.10	2	−0.15	2	25,200
5418.0	−1.06	1	−0.65	1	25,200
5546.1	−0.45	5	−0.18	5	25,200
5616.5	0	4	0	4	25,200
	+0.37	1	+0.50	1	25,200
5636.15	−0.22	6	−0.17	6	25,200
5637.6	−0.68	6	−0.76	6	25,200

REMARKS

λ 4130.62 Complex?
4133.94 Complex?
4198.62 *p*- and *n*-component displaced in different directions.

PLATE III



TAKAMINE—THE STARK EFFECT FOR METALS

In each illustration the *n*-components are above, the *p*-components below.

a	Iron	$E_{max.}$	= 29,800 volt/cm
b	Iron	$E_{max.}$	= 29,800 "
c	Nickel	$E_{max.}$	= 38,500 "
d	Molybdenum	$E_{max.}$	= 38,600 "
e	Molybdenum	$E_{max.}$	= 38,600 "
f	Nickel	$E_{max.}$	= 30,000 "

The line λ 3796.15 has a very faint detached component on the violet side, thus showing similarity to the copper lines λλ 3654.59 and 3688.60.

The series relation in the spectrum of gold is not known, but judging from their Stark effect the lines λλ 3796.15 and 4128.80 behave like diffuse series lines, while the line λ 4084.31 behaves like a sharp series line.

Cobalt.—Like nickel, cobalt shows a considerable number of affected lines. In the case of chromium, Anderson found that many of the affected lines were not given in the ordinary wave-length tables. For several lines in the cobalt spectrum this is also the case, but for reasons similar to those given by Anderson in his paper they are included in Table III. A group of affected lines near λ 4100 is shown in Plate II*e*.

Copper.—The Stark effect for copper lines is in many respects similar to that for silver. Although too faint to be seen in Plate II*f*, the diffuse series lines λλ 3654.59 and 3688.60 are accompanied by detached components on the violet side. A series of lines, evidently of Koch's type, was also observed in the copper spectrum. Their approximate wave-lengths are 3652.6, 3686.7, 4015.8, and 4056.8 A, the last two being superposed on the detached components of the diffuse series lines λλ 3654.59 and 3688.60.

The lines which form the term $n=4$ of diffuse series, namely λλ 4022.83, 4062.14, and 4063.50, are all displaced toward the red, as shown in Plate II*g*. On account of the great intensity, a number of ghosts appear on either side in the reproduction.

Iron.—As will be seen in Table V and Plate III*a*, *b*, the Stark effect for iron lines is exceedingly small compared with other elements, but the fact that it can be observed seems to be of no small importance, especially in connection with the pole effect, which will be discussed later.

In the present investigation, which is of a preliminary character, only the green portion of the iron spectrum was studied.

Magnesium.—In a previous investigation, carried out by N. Kokubu and the author,[1] four magnesium lines, λλ 3093.1, 3097.1,

[1] *Memoirs of the College of Science* (Kyoto Imperial University), **3**, 173, 1918.

TABLE IV

COPPER

λ	Series	p-Component		n-Component		$E_{max.}$ in Volt/cm
		Δλ	Int.	Δλ	Int.	
3654.59	IN, II6	V − 2.51	2	V −2.49	2	44,000
		r + 4.80	3	r +3.45	3	
3686.67	.....	+ 1.26	4	+1.40	4	44,000
3688.60	IN, I6	V_3 −10.30	1	.....	.....	44,000
		V_1	3	.....	..	
		o	2	o	2	
		r + 6.72	5	r +5.36	5	
3825.13	IIN, II5	− 0.16	3	−0.10	3	44,000
3830	.....	+ 1.79	2	+0.61	2	44,000
3861.88	IIN, I5	− 0.11	3	−0.12	3	44,000
3894.64	.....	+ 0.27	1	+0.23	1	44,000
4015.8	.....	− 0.47	2	−0.43	2	33,000
4022.83	IN, II5	+ 0.46	7	+0.33	7	33,000
4056.8	P2	− 0.62	3	−0.65	3	33,000
4062.14	IN, I5	+ 0.60	10	+0.44	10	33,000
4063.50	IN, I5	+ 0.37	4	+0.33	4	33,000
4480.59	IIN, II4	+ 0.09	6	+0.11	6	44,000
4531.04	IIN, I4	+ 0.04	6	+0.03	6	44,000
5153.33	IN, II4	− 0.10	6	−0.05	6	25,300
5218.45	IN, I4	− 0.09	10	−0.07	10	25,300
5220.25	IN, I4	.....	4	−0.06	4	.....

REMARKS

λ 3654.59 The violet component is detached from the main line in both the p- and n-components. Superposed on the end of this violet component is another line which appears only in strong electric field, having the wave-length 3652.0.

3688.60 In the p-components v_1 approaches the main line as the field-strength is increased.

4015.8 The line appears only in strong electric field, and may be regarded as a detached component of 4022.83.

4056.8 The line appears only in strong electric field and may be regarded as a detached component of 4062.14.

4352.2, and 4703.3, were found to be displaced toward the red, and the line λ 4571.3 toward the violet in an electric field. Besides these lines the strong triplet λλ 3829.5, 3832.5, and 3838.4 showed a tendency to shift toward the violet, but as the dispersion was very small (30 A per mm), the fact was stated with reserve at that

TABLE V

IRON

λ	p-COMPONENT		n-COMPONENT		$E_{max.}$ IN VOLT/CM
	Δλ	Int.	Δλ	Int.	
5133.675.....	−0.14	5	−0.15	5	29,800
5162.312.....	+0.73	4	+0.28	4	29,800
5324.196.....	+0.05	2	+0.05	2	29,800
5364.859.....	−0.06	2	.....	.....	29,800
5367.455.....	−0.13	3	−0.05	3	29,800
5404.131.....	−0.05	5	−0.05	3	29,800
5410.900.....	−0.02	3	−0.04	3	29,800
5415.189.....	−0.11	6	−0.06	6	29,800
5424.057.....	−0.13	10	−0.12	10	29,800
5445.040.....	−0.06	2	−0.06	2	29,800
5455.435.....	−0.35	8	−0.38	8	29,800

REMARKS

λ 5455.435 This line was not resolved from the neighboring line 5455.614.

time. In the present work, using a dispersion of about 5.3 A per mm, the results in Table VI were obtained.

TABLE VI

MAGNESIUM

λ	p-COMPONENT		n-COMPONENT		$E_{max.}$ IN VOLT/CM
	Δλ	Int.	Δλ	Int.	
3529.501.....	−0.08	3	−0.07	3	30,000
3832.450.....	−0.11	7	−0.06	7	30,000
3838.435.....	−0.12	10	−0.05	10	30,000

Molybdenum.—The spectrum of molybdenum shows many affected lines, especially in the green and violet regions. In the *p*- and *n*-components of the lines λλ 5238.41 and 5364.50 we notice the same complementary phenomenon in relative intensity which was noted in the case of the silver line λ 3810.85. In addition we have the curious feature that the *p*-component of λ 5238.41 behaves exactly like the *n*-component of λ 5364.50, while the *n*-component of the former behaves just like the *p*-component of the latter. This remarkable feature is illustrated in Plate III*d*, *e*.

TABLE VII

MOLYBDENUM

λ	p-COMPONENT		n-COMPONENT		$E_{max.}$ IN VOLT/CM
	Δλ	Int.	Δλ	Int.	
4017.50	−0.44	2	−0.18	2	38,600
4025.64	+0.41	1	+0.39	1	38,600
4028.85	−1.49	4	−1.06	4	38,600
4032.65	−1.51	3	−1.07	3	38,600
4060.4	−1.01	3	−0.62	3	38,600
4071.54	−1.17	1	−1.09	1	38,600
4157.58	0 +0.92 −0.47	2 4 3	0 +0.81	2 2	38,600
4284.79	v_2 −1.78 v_1 −0.13 0 r_1 +1.45	3 2 2 4	v −1.76 0 +0.18 r +1.86	3 3 2	38,600
4290.40	+0.47	3	+0.36	3	38,600
4529.53	r_1 +0.41 r_2 +2.09	2 1	v −0.34 0 r +1.53	1 2 2	38,600
4535.09	+0.53	3	+0.58	3	38,600
4567.87	+0.20	1	+0.22	1	38,600
4582.69	−0.89 0	3 2	−0.49	2	38,600
4652.49	+0.42	4	+0.33	3	38,600
4688.41	−0.20	3	−0.20	3	38,600
4758.71	−0.13	2	.....	.....	38,600
4764.64	+0.25	3	+0.25	3	38,600
5171.33	+0.49	4	+0.94	2	38,600
5173.14	−0.13	5	−0.04	5	38,600
5174.35	−0.33	6	−0.33	6	38,600
5238.41	−1.48	10	−1.41 +0.34	3 4	38,600
5241.09	.....	.....	−0.50 0 +0.47	2 1 1	38,600
5243.01	0 +1.07	2 3	+0.32	3	38,600
5245.71	−0.10	4	.....	.....	38,600
5260.76	+0.81	10	+0.64	10	38,600
5264.50	−1.22 +0.43	2 4	−1.40 +0.43	5 1	38,600
5367.30	−1.22 0 −0.48	2 1 1	+0.48 −0.60 0 +0.57	1 1 1 1	38,600
5406.64	+0.49	1	+0.35	1	38,600
5544.78	+0.34	1	+0.49	1	38,600

REMARKS

λ 5241.09 This line may be regarded as a detached component of 5238.41. The *p*-component is too faint for measurement.

5367.30 This line may be regarded as a detached component of 5364.50. The *n*-component is very weak.

Nickel.—In 1917 Anderson[1] observed a number of affected nickel lines near λ 4000 A. In the present work, using a stronger electric field, the effect was much enhanced, as shown in Plate III*f*. Many affected lines were also found in the green and yellow regions. Plate III*c* shows the effect for λ 5084.20.

The Stark effect for a few lines of nitrogen, sodium, and oxygen was observed rather accidentally in the course of the present work.

Nitrogen.—In our previous paper dealing with the Stark effect of calcium and magnesium[2] it was reported that certain lines of unknown origin came out quite frequently when Al, Ca, Mg, and Ta were used as the cathode, the residual gas being air or hydrogen. Two of these lines, namely λλ 4100.7 and 4110.3, were found to be displaced toward the violet and were marked by the letters *C* and *D* in our paper. Later, Yoshida noticed that these lines had been recorded by Moissan and Deslandres[3] in their investigation of the nitrogen spectrum.

In his study of the Doppler effect in canal rays, Hermann[4] mentions that the two nitrogen lines λλ 4100 and 4110 behave differently from other lines. Since the discharge in pure nitrogen was not tried in the present work, the origin of these two lines may still be open to question. Table IX gives the displacement of these two lines.

Sodium.—The cobalt cubes used contained a small quantity of sodium as impurity, which brought out the sodium lines λλ 5682.90 and 5688.26 fairly strongly. They belong to the first subordinate series of sodium and were found to be displaced toward the red, as shown in Table X.

[1] *Physical Review*, **9**, 575, 1917.

[2] T. Takamine and N. Kokubu, *Memoirs of the College of Science* (Kyoto Imperial University), **3**, 173, 1918.

[3] *Comptes Rendus*, **126**, 1689, 1898.

[4] *Physikalische Zeitschrift*, **7**, 567, 1906.

TABLE VIII

Nickel

λ	p-Component		s-Component		$E_{max.}$ in Volt/cm
	Δλ	Int.	Δλ	Int.	
3826	+0.69	3	+0.57	3	.....
3831.89	+0.27	1	+0.17	1	.....
3893	−0.96	2	−0.83	2	.....
3896	−0.82	2	−0.82	2	.....
3909.10	+0.17	3	+0.18	3	.....
3918.7	−0.17	1	−0.23	1	39,000
3921.7	−0.93	4	−0.93	4	39,000
3939.3	+0.49	3	+0.57	3	39,000
3941	−1.01	3	−1.02	3	39,000
3944.3	0	2	0	2	39,000
	+0.39	7	+0.40	7	39,000
3953.0	+0.65	1	+0.64	1	39,000
3954	−1.21	4	−1.21	4	39,000
3955.2	+0.57	1	+0.47	1	39,000
3957.8	+0.62	1	+0.57	1	39,000
3960.7	−1.36	1	.....	1	39,000
3962.25	−1.16	4	−1.22	4	39,000
3964.4	+0.54	3	+0.42	3	39,000
3968.3	−0.65	1	−0.51	1	39,000
3970.65	+0.19	8	+0.25	8	39,000
3972.0	+0.40	3	+0.40	3	39,000
3974.83	+0.48	6	+0.44	6	39,000
3984.18	−1.46	7	−1.31	7	39,000
3987.2	+1.12	2	+1.06	2	39,000
3994.1	+0.50	5	+0.57	5	39,000
3998.5	+0.36	2	+0.36	2	39,000
3999.8	+0.18	2	+0.18	2	39,000
4007.8	−0.77	1	−0.73	1	39,000
4017.67	−1.04	8	−0.91	8	39,000
4022.5	−1.00	2	−1.00	2	39,000
4024.1	−0.48	3	0	2	39,000
	0	.....	.....	.....	39,000
4024.9	+0.99	2	+0.59	2	39,000
4025.1	+0.42	4	+0.41	4	39,000
4025.8	−0.70	3	−0.52	3	39,000
4027.8	−1.54	5	−1.44	5	39,000
4029.9	+0.70	2	.....	.....	39,000
4034	+0.27	2	+0.24	2	39,000
4037.7	+0.96	3	+0.70	3	39,000
4410.66	−0.18	6	−0.08	6	39,000
4937.45	−1.21	1	−1.34	1	38,500
5018.48	+0.70	2	.....	.....	.....
5082.55	+0.44	3	+0.25	3	38,500
5084.20	v −1.29	7	−1.40	4	38,500
	0	1	0	.....	
	r_1	.....	r_1 +1.92	.....	
	r_2 +4.70	1	r_2 +4.62	1	

TABLE VIII—*Continued*

λ	*p*-Component		*n*-Component		$E_{max.}$ in Volt/cm
	Δλ	Int.	Δλ	Int.	
5100.12.....	+0.30	8	+0.15	8	38,500
5142.91.....	−0.09	4	−0.07	4	21,800
5146.61.....	−0.13	3	−0.07	3	21,800
5155.90.....	−0.79	5	−0.79	5	21,800
5176.72.....	−0.15	4	−0.09	4	21,800
5184.78.....	+0.29	3	+0.30	3	21,800
5462.69.....	−0.86	1	−0.56	1	21,800
5588.09.....	−0.15	2	−0.11	2	21,800

REMARKS

λ 3893 Band superposed on this line.
3909.10 Band superposed on this line.
3939.3 Koch's type.
3957.8 Koch's type.
3962.25 Koch's type. Detached component of 3962.25?
3987.2 Koch's type.
4024.9 Detached component of 4024.1?
5084.20 In both *p*- and *n*-components r_1 and r_2 are detached.

TABLE IX

Nitrogen

λ	*p*-Component		*n*-Component		$E_{max.}$ in Volt/cm
	Δλ	Int.	Δλ	Int.	
4100.7.....	−0.15	5	−0.27	5	56,000
4110.3.....	−0.50	5	−0.40	5	56,000

TABLE X

Sodium

λ	Series	*p*-Component		*n*-Component		$E_{max.}$ in Volt/cm
		Δλ	Int.	Δλ	Int.	
5682.90.....	IN, II4	+1.82	2	+1.72	2	25,200
5688.26.....	IN, I4	+2.21	2	+2.08	2	25,200

Oxygen.—The red oxygen lines λλ 6156.198, 6156.993, and 6158.415, which form a triplet in the first subordinate series, were all found to be displaced toward the red, as shown in Table XI. It may be remarked here that in the discussion given by Schönrock[1] on the width of lines, the data for this triplet show considerable discrepancy, which may perhaps be accounted for by the Stark effect.

In connection with this it may be mentioned that the Stark effect for a number of green oxygen lines was first observed by Yoshida.

TABLE XI

OXYGEN

λ	Series	*p*-Component		*n*-Component		$E_{max.}$ in Volt/cm
		Δλ	Int.	Δλ	Int.	
6156.198.....	TNI	+0.37	1	+0.48	1	18,200
6156.993.....	TNI	+0.51	2	+0.39	2	18,200
6158.415.....	TNI	+0.60	3	+0.39	3	18,200

THE STARK EFFECT AND THE POLE EFFECT

One of the interesting features revealed by the present investigation is that there seems to be an unmistakable connection between the pole effect and the Stark effect. The pole effect for different elements has been studied by various investigators, an elaborate study of the phenomenon having been specially made for iron by St. John and Babcock.[2] Table XII gives the comparison of these two effects for iron lines.

As will be seen from the table, the two effects are in complete agreement as to the direction of displacement. Quantitatively, also, the agreement is fairly good except for the three lines λλ 5162.312, 5424.057, and 5455.435. The last line is a blend, made up of λ 5455.614, which is an "*a*" line in the classification of St. John and Babcock, and λ 5455.435, which is an "*e*" line, and

[1] *Annalen der Physik*, 20, 995, 1906.

[2] *Mt. Wilson Contr.* Nos. 106, 137; *Astrophysical Journal*, 42, 231, 1915; 46, 138, 1917.

TABLE XII

IRON

λ	POLE EFFECT	STARK EFFECT ($E_{max.}$: 29,800 VOLT/CM)	
		p-Component	*n*-Component
5133.675	−0.045	−0.14	−0.15
5162.312	+0.030	+0.73	+0.28
5324.196	+0.015	+0.05	+0.05
5364.859	−0.028	−0.06	.....
5367.455	−0.025	−0.13	−0.05
5404.131	−0.025	−0.05	−0.05
5410.900	−0.026	−0.02	−0.04
5415.189	−0.030	−0.11	−0.06
5424.057	−0.027	−0.13	−0.12
5445.040	−0.020	−0.06	−0.06
5455.435	−0.016	−0.35	−0.38

hence shifted to the violet in the pole effect. In my spectrograms the dispersion was insufficient to separate the two, and consequently the displacement given is subject to considerable error.

The pole effect for nickel lines was recently examined by Babcock and Merrill (unpublished), and a comparison of pole and Stark effect was also made for this element. In the green part of the spectrum very close agreement was found, as shown in Table XIII; but in the violet part those lines which show Stark effect are so diffuse in the pole effect spectrograms, especially at the negative pole, as to render the measurement almost impossible. An inspection of these diffuse lines, however, indicates that most of them are widened toward the same side as in the case of the Stark effect.

TABLE XIII

NICKEL

λ	POLE EFFECT	STARK EFFECT		$E_{max.}$ IN VOLT/CM
		p-Component	*n*-Component	
4937.45	−0.006	−1.21	−1.34	38,500
5018.48	+0.006	+0.70	.....	38,500
5082.55	+0.009	+0.44	+0.25	38,500
5084.20	−0.009	−1.29	−1.40	38,500
5100.12	+0.015	+0.30	+0.15	38,500
5155.90	−0.007	−0.79	−0.79	21,800
5176.72	−0.008	−0.15	−0.09	21,800
5184.78	+0.011	+0.29	+0.30	21,800

It should be remarked that in discussing the various causes of the broadening of spectrum lines Stark[1] suggested that the pole effect may be due to intermolecular electric fields. On comparing the results for the Stark effect with those obtained by King[2] for furnace spectra, it was found that the lines which are affected in an electric field belong to high-temperature lines in King's classification.

A preliminary search for the possible relations between the Stark effect and the pressure effect on spectrum lines was made also. For this purpose a series of investigations on the pressure effect of the lines of Ag, Au, Cu, Fe, and Ni carried out by Duffield[3] was referred to. Although it is difficult to make any definite statement, still in many cases there seem to be some indications that the Stark effect is involved as one of the causes affecting the pressure shift. For example, in the case of copper, Duffield classifies the lines in his Table VI in two groups, the one being strengthened as the pressure increases, while the other is weakened. On referring to the Stark effect we find that the lines which are affected by an electric field are all included in the latter class. In comparing the pressure effect with the Stark effect, however, we see many lines which are broadened on one side with high pressure, but remain unaffected by an electric field, so that the relation is not so simple as in the case of pole and Stark effect.

The relation between the Stark effect and the broadening of lines in arc and spark spectra was studied by Stark and Kirschbaum,[4] Stark,[5] Wendt,[6] Merton,[7] and others. In the course of the present work the following features were noticed:

1. In the ordinary arc spectrum of copper the lines λλ 4022.83 and 4063.50 are usually very diffuse toward the red. In the Stark effect these two lines are also displaced toward the red.

2. Striking similarity was noticed in the appearance of the silver lines λλ 4210.87 and 4212.76 as they appear in the arc and in a

[1] *Jahrbuch der Radioaktivität*, **12**, 349, 1915.

[2] *Mt. Wilson Contr.* Nos. 66, 108; *Astrophysical Journal*, **37**, 239, 1913.

[3] *Trans. Royal Soc.*, **208A**, 111, 1908; **209A**, 205, 1908; **211**, 33, 1909; **215**, 205, 1915.

[4] *Annalen der Physik*, **43**, 1017, 1914.

[5] *Jahrbuch der Radioaktivität*, **12**, 349, 1915.

[6] *Annalen der Physik*, **45**, 1257, 1914. [7] *Loc. cit.*

strong electric field. (Compare Fig. 1, Plate I, in Duffield's paper, *Phil. Trans.*, **211 A,** 33, 1909, with Plate II*c*.)

3. Generally speaking, those diffuse lines which are marked by the letters *u* and *U* in tables of wave-length given in Kayser's *Handbuch der Spectroscopie* were found to be affected by an electric field.

4. In the green part of the spectrum of a condensed spark discharge between iron poles those lines which are affected by an electric field appear invariably very hazy.

STARK EFFECT AND SERIES RELATIONS

Unfortunately the series relations in the spectra of the elements investigated here are not yet known except for silver and copper.

In Table XIV the affected lines of silver and copper are arranged according to their series relation, the + and − signs showing the direction of displacement in an electric field.

TABLE XIV

	Silver		Copper	
	I.N.S.	II.N.S.	I.N.S.	II.N.S.
$n=4$	5471.72? 65.66? 5209.25?	4668.70+ 4476.29+	5220.25− 18.45− 5153.33−	4531.04+ 4480.59+ .....
$n=5$	4212.76 (complex) 4210.87 (complex) 4055.46 (complex)	3981.87+ 3840.74+	4063.50+ 62.14+ 22.83+	3861.88− 25.13− .....
$n=6$	3810.85 (complex) 3682.45 (complex)	3710.1 + ...	3688.60 (complex, main line+) 54.59 (complex, main line+)	

In general, the diffuse series lines (I.N.S.) show more or less complex decompositions, while the sharp series lines (II.N.S.) are simply displaced in one direction, a feature which is very pronounced in the Stark effect for helium lines.[1]

[1] The fact that some of the sharp series lines of helium are not merely shifted in an electric field, but are resolved into a number of components, is stated in a previous paper (*Proceedings Tokyo Math. Phys. Soc.*, II, **9**, 394, 1918).

With respect to the direction toward which the lines are displaced, silver shows all its sharp series lines displaced toward the red, while in the case of copper the lines of both sharp and diffuse series change sign of displacement as we go from the term number $n=4$ to $n=5$. It is remarkable that exactly the same feature was observed for the diffuse-series lines of magnesium in our previous work,[1] in that the lines λλ 3829.5, 3832.5, and 3838.4 corresponding to $n=4$ are displaced toward the violet, while the members of the next term λλ 3093.1 and 3097.1 are displaced toward the red.

In general the relation of the Stark effect to the series law seems to be less simple than that of the Zeeman effect. An extension of our study of the Stark effect to the ultra-violet region seems to be highly desirable.

For those elements whose series relations are not yet established it appears that the Stark effect may be utilized as a powerful means of finding the relationships.

It may be worth mentioning that, of the elements studied in the present work, Na, Cu, Ag, and Au are all in the first column of the periodic table, while O and Mo together with Cr, which was studied by Anderson, are in the sixth column. The similarity of the electric decomposition of the lines of the elements in any one column of the periodic table is very striking when we compare, for example, the lines of Ag, Au, and Cu; also those of Mo and Cr as well as those of Fe, Co, and Ni.

In conclusion I wish to express to the members of the Observatory staff my very deep appreciation of the opportunity for my work at the Observatory. I am under particular obligation to Dr. J. A. Anderson for his kind guidance throughout the experiments, and to Miss M. O. Burns of the Computing Division for her careful measurements and reductions.

SUMMARY

1. Adopting the method employed by Anderson, the Stark effect on the spectra of the following metals was investigated: Ag, Au, Co, Cu, Fe, Mg, Mo, Ni. In addition to this a few lines of Na, O, and N were found to be affected by an electric field.

[1] T. Takamine and N. Kokubu, *loc. cit.*

2. Close relations were found between the pole effect and Stark effect in the spectra of iron and nickel.

3. Several instances showing the close relation between the Stark effect and the broadening of lines in the arc and spark spectra were noticed.

4. The connection between the series relationship and the Stark effect seems to be somewhat more complicated than in the case of the Zeeman effect. In some cases the side toward which the line is displaced in an electric field is different for different term numbers.

5. As in the case of helium, detached components and isolated lines, having the peculiar property of showing themselves only in strong electric fields, were observed in the spectra of silver and copper.

6. In the spectrum of molybdenum a curious complementary feature between *p*- and *n*-components of the two lines, λλ 5238.41 and 5364.50, was observed in that the *p*-component of the former was exactly the same as the *n*-component of the latter, and vice versa.

MOUNT WILSON OBSERVATORY
March 1919

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

THIRTEENTH PAPER: THE GALACTIC PLANES IN 41 GLOBULAR CLUSTERS

By HARLOW SHAPLEY and MARTHA B. SHAPLEY

From evidence presented in earlier papers there can be little doubt that the observed elliptical distribution of stars in the photographic images of globular clusters is really an indication of flattening with respect to more or less symmetrical equatorial planes. The analogy with other flattened sidereal organizations—such as the Galaxy, the spiral nebulae, the solar and planetary systems—leads us to interpret these projected elongations of clusters as indicative not of prolate spheroids but of oblate spheroids or ellipsoids. It is of some interest to know how general this phenomenon is among globular clusters; and since their relation to the galactic system has been demonstrated, it is important to see if their orientation in space betrays any effect of the dominance of the greater system.

A study of star counts by Pease and Shapley[2] has shown the elliptical form for five clusters and has suggested that three others are sensibly circular. The ellipticity of Messier 5 has also been determined from Mount Wilson plates.[3] Further discussion by Shapley[4] has indicated the relation of the elongation in ω Centauri to its variable stars, and in Messier 13 to the stars of bluish color. The present paper continues the work, increasing the number of clusters known to have appreciable elongation from six to about thirty, and showing that several others are sensibly circular, either

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 160.

[2] *Mt. Wilson Contr.* No. 129; *Astrophysical Journal*, **45**, 225, 1917; *Mt. Wilson Communications*, No. 39; *Proceedings of the National Academy of Sciences*, **3**, 96, 1917.

[3] Shapley and Davis, *Publications of the Astronomical Society of the Pacific*, **30**, 164, 1918.

[4] *Mt. Wilson Communications* No. 45; *Proceedings of the National Academy of Sciences*, **3**, 276, 1917.

because of their orientation in space or because of actual sphericity. All globular clusters sufficiently bright (and in other respects satisfactory) to give definite results have been investigated. The accumulated material, though not extensive, is complete enough to permit statistical examination.

TABLE I

COMPARISON OF COUNTS AND ESTIMATES

N.G.C.	MESSIER	POSITION-ANGLE OF MAJOR AXIS		
		Counts	Estimates	Difference
5024	53	160°	170°	−10°
5139	.....	105	120	−15
5272	3	Asym.	Asym.	.....
5904	5	55	50	+ 5
6121	4	115	Indeterm.	.....
6205	13	125	130	− 5
6218	12	Appr. circ.	Appr. circ.	.....
6254	10	See note	Appr. circ.	.....
6266	62	75	65:	+10:
6273	19	15	15	0
6341	92	Indeterm.	25	.....
6402	14	110	70:	+40:
6626	28	50	45	+ 5
6656	22	25	25	0
6779	56	Appr. circ.	150:	.....
7078	15	35	20	+15
7089	2	135	150	−15

N.G.C. 5139 (ω Centauri). The direction of the major axis, as estimated from photographs made at Harvard, is 105°.

N.G.C. 5272. Cf. tabulated counts in *Mt. Wilson Contr.* No. 129.

N.G.C. 5904. Cf. discussion of asymmetry in *Publications of the Astronomical Society of the Pacific*, **30**, 164, 1918.

N.G.C. 6205. The image of the Hercules cluster on the Franklin-Adams chart is too distant from the center and too diffuse to permit dependable estimates. The tabulated value is from photographs by Pease.

N.G.C. 6254. Perhaps the counts given in *Mt. Wilson Contr.* No. 129 show a trace of ellipticity, major axis in position-angle 150°.

The Franklin-Adams charts, which were of much value in determining the relative diameters of globular clusters, are also highly useful in the present problem. Although the ellipticity of clusters cannot be accurately determined from ordinary photographs without a special study of the distribution of the individual stars, and generally then only when many faint stars are included in the

counts, it is found that direct estimates based upon photographs of small scale give values of the direction of elongation as definite as those resulting from the extensive counts of stars. The direct estimates do not give the degree of elongation, but that property is also indefinite from counts, for it apparently depends in an uncertain manner upon the brightness of the stars enumerated and upon other factors.

In Table I the direction of elongation, when measurable, is given for 17 clusters, both from estimates and counts, the two determinations being entirely independent of each other. The average difference[1] between the position-angles thus derived is 8°. Better agreement than this is not obtainable from counts of two plates of the same cluster, or from the counts on one plate of stars of different magnitudes, colors, or distances from the center.[2] The consistent agreement therefore justifies the substitution of direct estimates for laborious counts. We are therefore enabled to extend the investigation expeditiously to the bright southern objects and complete the survey of the form of globular clusters.

Ten of the values of the third column of Table I are from published results. Those for N.G.C. 6121, 6266, 6273, 6341, 6402, 6626, and 6656, based on Mount Wilson plates, are from recent counts made by Miss Davis subsequent to the estimations from the charts. New counts have also been made of the stars in N.G.C. 5139 (ω Centauri) on plates kindly lent by the Harvard College Observatory. Some of the more interesting results of the unpublished counts are noted below.

1. The decided asymmetry of N.G.C. 6266 (Messier 62), particularly noted by Sir John Herschel[3] and later by Bailey,[4] is shown by the following values derived from a Mount Wilson plate:

Sector.....	15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°
No. stars....	67	56	67	45	35	42	49	56	68	79	72	72

[1] Omitting N.G.C. 6402, which is very open and ragged in outline on the Franklin-Adams chart.

[2] Cf. curves for Messier 13 in *Mt. Wilson Contr.* No. 129.

[3] *Cape Results* (London, 1847), p. 23.

[4] *Harvard Annals*, 76, 74, 1915.

Stars within 30″ of the center could not be counted. Combining opposite sectors, the position-angle of the major axis appears to be about 75°.

2. Probably the most conspicuously elongated globular cluster is N.G.C. 6273 (Messier 19), whose relative stellar density is shown in Fig. 1. The radial co-ordinate is number of stars for each 30° sector of position-angle. Without correction for superposed field stars, there are more than twice as many stars in the direction of the major axis as perpendicular thereto. The plates show no evidence of a double nucleus. We probably see this flattened cluster more nearly edgewise than is usually the case. The major axis is approximately parallel to the Milky Way, the galactic latitude is +9°, and it appears likely, therefore, that the plane of symmetry is nearly parallel to the galactic plane. Apparently the flattening of a cluster, if we judge from this extreme case, is of a much lower order than that of spirals and of the galactic system.

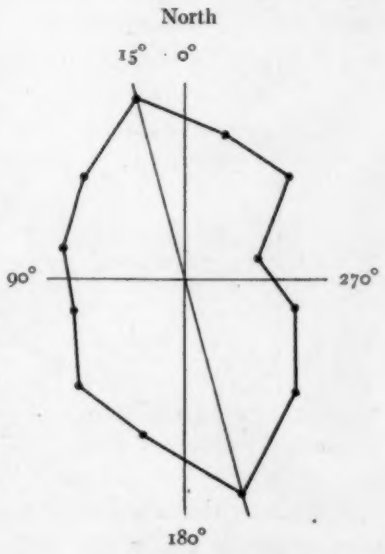


FIG. 1.—Curve of star-density for Messier 19. Radial co-ordinates show the number of stars for each 30° of position-angle.

3. The cluster N.G.C. 6626 (Messier 28) is also conspicuously elongated, with major axis roughly parallel to the Milky Way, but it is hardly as symmetrical as Messier 19.

4. A photograph made with the 60-inch reflector of the bright southern cluster N.G.C. 6656 (Messier 22), which shows more than 70,000 stars, has been counted by Miss Davis.

Nearer the center than 8 mm (218″) the density is too great to give reliable results; under such conditions the large images of the brighter stars occult so many faint images that counts

of the latter may show the ellipticity obliterated or even reversed.

Outside the circle of 14 mm (381″) radius the proportion of field stars becomes so large that the amount of ellipticity is masked, and its observed direction may be affected by local irregularities in the field. Neither of these factors—the interior crowding or the exterior field—is troublesome in estimating the direction of elongation of bright clusters from small-scale plates.

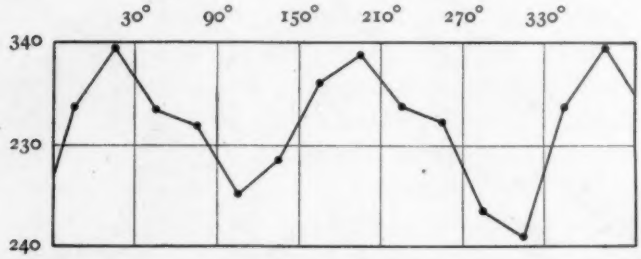


FIG. 2.—Distribution of stars in Messier 22. Ordinates are numbers of stars; abscissae are position-angles.

Counts within the limits 8 and 14 mm give the distribution of stars shown in Fig. 2, the ordinates being numbers of stars in 30° sectors and the abscissae position-angles. A correction for superposed field stars has been made by deducting 30 stars per square minute. Combining opposite sectors we have the following means for the smooth ellipticity-curve shown in Fig. 3:

Sector	15°	45°	75°	105°	135°	165°
No. stars	336	308	300	262	263	314

The number of stars in the direction of the major axis exceeds the number in the direction of the minor axis by nearly 30 per cent of the latter.

Within the errors of measurement the major axis of Messier 22 is exactly parallel with the Galaxy. The high ellipticity suggests that we see the cluster edgewise, and, since it is in low galactic latitude, the result further indicates that the plane of symmetry is

approximately parallel with the galactic plane. This point may be significant in connection with the loose structure of the cluster and its proximity to the central plane of the galactic system.

Table II contains data for 41 globular systems. When the direction of the major axis has been derived by two methods a weighted mean is adopted. The inclination of the observed major axis to the galactic plane, in the sixth column, has been computed from the data of preceding columns and checked graphically. The linear distances from the galactic plane, $R \sin \beta$, expressed in units of 100 parsecs, are taken from Tables V and VIII of *Mt. Wilson Contr.* No. 152.

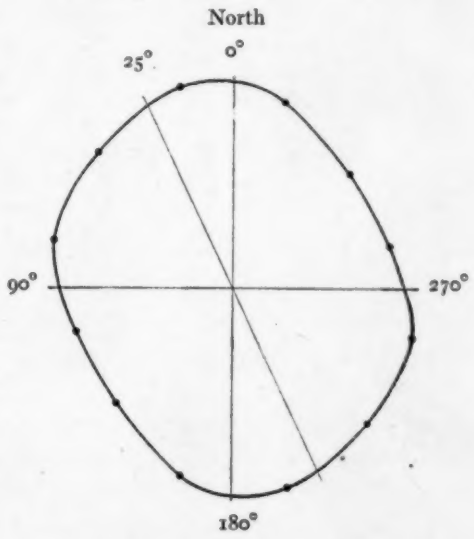


FIG. 3.—Curve of star-density for Messier 22. The radial co-ordinates show the number of stars for each 30° of position-angle.

In order that the equatorial plane of a cluster and the galactic plane may be parallel, the parallelism of the major axis with the galactic circle is a necessary though not sufficient condition. Without knowing another dimension of the inclination we can say little as to the possible influence of the greater system upon the orientation of its secondaries. There are 15 clusters whose major axes are inclined less than 35° to the galactic circle; some of these may be parallel with the Galaxy, but none of the others can be considered as even approximately so.

If the equatorial planes of globular clusters are distributed at random we should find both large and small inclinations of the major axes at all distances. This condition holds for the 25 clusters more distant than 2000 parsecs from the plane, their inclinations

ranging from 0° to 85° with an average of 45°, as shown in Table III. The major axes of all the 8 clusters within 3000 parsecs of the plane,

TABLE II

RESULTS FOR 41 CLUSTERS

N.G.C.	Messier	Galactic Longitude	Galactic Latitude	Adopted Position-Angle of Major Axis	Inclination of Major Axis to Galactic Circle	Distance from Galactic Plane (Unit is 100 Parsecs)
104	.....	272°	−44°	35°	45°	− 47
362	.....	268	−46	110	15	−109
1851	.....	211	−34	Appr. circ.	.....	− 96
1904	79	195	−28	20	30	−120
2808	.....	249	−11	20	65	− 32
4372	.....	269	− 9	30:	65:	− 18
5024	53	307	+79	165	75	+186
5139	.....	277	+16	110	30	+ 18
5272	3	8	+77	Asym.	.....	+135
5286	.....	279	+11	Appr. circ.	.....	+ 37
5904	5	333	+45	55	15	+ 88
5986	.....	305	+13	Appr. circ.	.....	+ 47
6093	80	320	+18	165	55	+ 62
6121	4	319	+15	115	70	+ 30
6171	.....	331	+22	65:	30:	+ 60
6205	13	26	+40	125	65	+ 71
6218	12	344	+25	Appr. circ.	.....	+ 52
6254	10	343	+22	Appr. circ.	.....	+ 45
6266	62	320	+ 7	70	35	+ 19
6273	19	324	+ 9	15	20	+ 25
6284	.....	325	+10	Appr. circ.	.....	+ 64
6293	.....	325	+ 8	35	0	+ 37
6304	.....	323	+ 5	Appr. circ.	.....	+ 28
6341	92	35	+34	25	15	+ 69
6362	.....	293	−17	110	80	− 38
6397	.....	304	−12	35	5	− 17
6402	14	349	+14	100	70	+ 56
6541	.....	316	−11	45	20	− 28
6584	.....	309	−16	Appr. circ.	.....	− 73
6626	28	336	− 7	45	20	− 23
6656	22	338	− 9	25.	0	− 13
6681	70	329	−13	150:	55:	− 41
6715	54	333	−16	95	70	− 44
6723	.....	327	−18	Appr. circ.	.....	− 39
6752	.....	303	−26	125:	65:	− 39
6779	56	30	+ 7	Appr. circ.	.....	+ 30
6809	55	335	−24	100	80	− 41
7006	.....	32	−20	115	85	−228
7078	15	33	−29	30	10	− 71
7089	2	22	−37	140	70	− 94
7099	30	356	−48	20	5	−128

however, are inclined less than 35°, except for N.G.C. 4372, which is very loose and apparently is partially obstructed by dark nebulosity.[1]

[1] Cf. p. 22, n. 2, of *Mt. Wilson Contr.* No. 152.

There is some evidence, therefore, that the equatorial planes in clusters may be roughly parallel to the galactic plane when near it, but that when distant they are oriented at random in space.

TABLE III

AVERAGE INCLINATION TO GALAXY FOR DIFFERENT DISTANCES FROM GALACTIC PLANE

Interval of $R \sin \beta$	Number of Clusters	Mean $R \sin \beta$	Mean Inclination
0 to ±20.....	5	17	23°
±20 to ±40.....	8	31	41
±40 to ±60.....	5	46	64
±60 to ±80.....	5	67	36
>±80.....	7	136	42
>±20.....	25	72	45°

SUMMARY

1. The good agreement of results from counts on large-scale photographs with results from estimates on Franklin-Adams charts justifies the use of the latter for detecting the ellipticity of clusters (Table I).

2. Thirty globular clusters show an apparent elongation due to the discoidal distribution of their stars; a number of others appear circular in outline, due either to actual sphericity or, more probably, to the high inclination of their planes of symmetry to the line of sight (Table II).

3. Probably the most elongated cluster on record is Messier 19, for which the stars are more than twice as numerous in position-angles 15° and 195° as in directions perpendicular to the major axis of the projected ellipse (Fig. 1).

4. The flattening of a globular cluster, while appreciable, is obviously very small in comparison with that of spiral nebulae or with what now appears to be the form of the general galactic system.

5. The equatorial plane of the globular cluster nearest the galactic plane, Messier 22, appears to be parallel to the Galaxy (Figs. 2 and 3).

6. In general, the clusters nearest the dense stellar regions are most nearly parallel to the galactic plane.

MOUNT WILSON SOLAR OBSERVATORY
September 1918

SPECTRAL SERIES AND THE GROUPING OF THE ASTEROIDS

By F. E. KESTER and DINSMORE ALTER

Within the region in the solar system between the orbits of Mars and Jupiter is scattered a swarm of small planetary bodies, known as the asteroids, of which more than eight hundred, ranging in size from a few miles to a few hundred miles in diameter, have been discovered. They are too small and too much scattered throughout this region to influence each other's motion perceptibly, except in possible rare cases. They are near enough Jupiter and that planet is large enough to make the perturbations dué to it far larger than those due to any other planet.

It is a fact well known by astronomers that the perturbations of the asteroids produced by Jupiter drive the asteroids from regions of commensurability of frequencies of revolution into regions of incommensurability. This effect is stronger the simpler the ratio of the frequencies of Jupiter and the asteroid.

While it is natural to think in terms of the frequencies of revolution, it will be convenient to work instead in terms of a regularly tabulated orbital element, namely the mean daily motion of the radius vector measured in seconds of arc. In Fig. 1 the horizontal lines are divided to represent such mean daily motions, and an ordinate is erected for each second of this motion to such height as to represent the number of asteroids whose mean daily motions have that particular value. The values are taken, to the nearest second in each case, from the *Berliner Jahrbuch* of 1916 for all asteroids which have been observed at more than one opposition, and for no others.

On this plot there are represented also the ratios of commensurability spoken of above. The asteroids are evidently affected by commensurabilities to the order of six revolutions of Jupiter to an integral number of revolutions of the asteroid, but seem not to respond with equal certainty to more complex ratios.

Much of the recent theoretical work which has been done on atomic structure indicates that there may be some analogy between the solar system and the atom, and suggests that possibly the

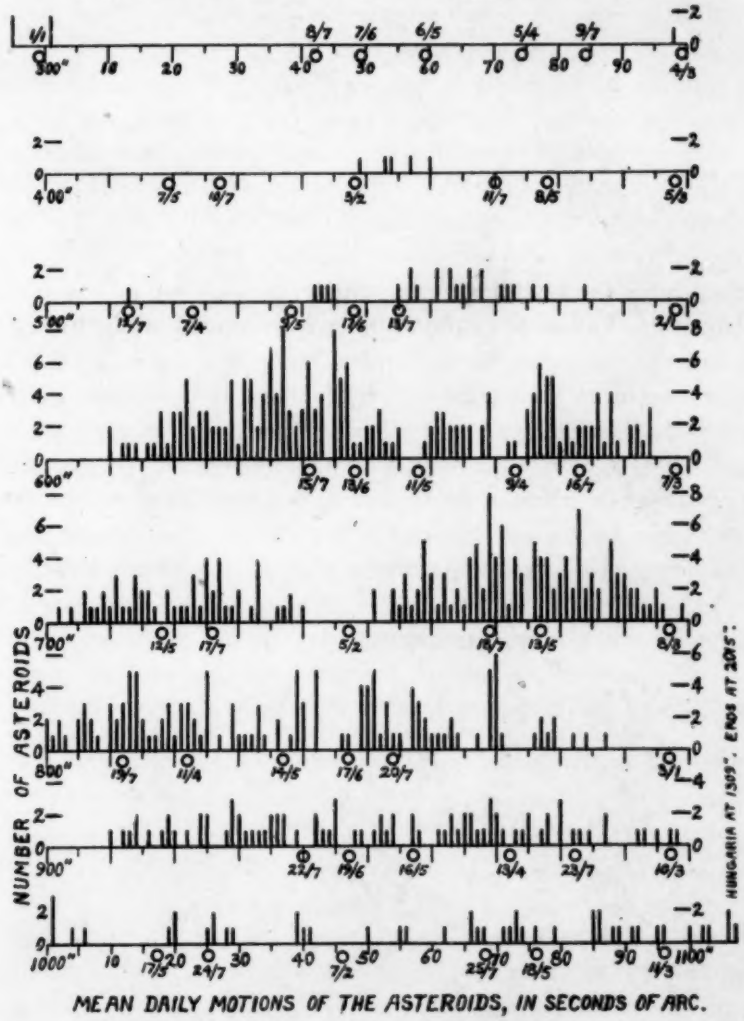


FIG. 1

grouping of orbits of electrons, as assumed by Bohr[1] to explain spectral series without any mechanical explanation therefor, may be due to perturbations in some manner similar to those that

[1] *Philosophical Magazine* (6), 26, 1–25, 476–502, 857–875, 1913.

Jupiter produces on the asteroids. An examination of all the asteroids for which the data may be considered sufficiently reliable has been made with this possibility in view.

Difficulties result from two causes as follows: (*a*) Near Jupiter and also for frequencies greater than 1100″ of mean daily motion the number of asteroids is too small to allow one to draw any definite conclusion. In these regions the positions may be represented by several equations of the Rydberg type without any certainty as to which is true. For this reason no representation of these regions has been considered worth publishing. The same thing is true, for a different reason, in the region near the converging frequency of the Rydberg series which we shall use in this paper. Between 850″ and 1100″ the number of asteroids is much smaller than in the preceding three hundred and fifty seconds. Certain maxima seem to appear in this region, but they are not definite enough to be picked out by all observers and therefore are not listed. The reader will notice that in the main such indefinite maxima may be represented by adding Jupiter's mean daily motion to the series tabulated herewith. (*b*) The mean daily motions of all the asteroids are constantly changing, due to perturbations. Elements osculating for the same date for all the asteroids, or preferably mean elements, are needed. The confusion due to this cause amounts probably to three or four seconds, making the observed maxima uncertain by that amount.

TABLE I

I	II	III	IV
455″	458″	.....	O–C −3″
563″	.....	562″	+1″
635″	630″	.....	+5″
677″	.....	677″	0″
711″	710″	.....	+1″
729″	.....	734″	−5″
	852″ converging frequency		.....

Column I of Table I shows the observed maxima of the asteroid groups. In estimating maxima great care has been taken to be as conservative as possible, and to include no maxima which the

plot of mean daily motions fails to show as certain. To this end the judgments of a number of observers have been taken. The reader must judge for himself how well this has been done.

The first series attempted was that discovered by Balmer for spectrum lines, which for the present purpose may be put into the Rydberg form

$$n=N\left(\frac{1}{p^2}-\frac{1}{q^2}\right),$$

where, for the Balmer series, $p=2$. This proved to be the only one whose presence we could certainly demonstrate. With $p=2$ a fair representation of a number of groups of asteroids was secured. It was then noted that the value used for the constant N was very nearly that of the frequency-constant of the solar system, namely the value of k in the formula

$$n=k/a^{3/2},$$

in which n is the mean daily motion as defined above and a is the semimajor axis of the planet measured in terms of that of the earth. This value was at once substituted and a solution made to obtain a more exact value of p. This turned out to be 2.041. With these values the formula gave a better representation than at first trial.

Columns II and III give maxima computed from the equations

$$n=3548\left[\frac{1}{(2.041)^2}-\frac{1}{m^2}\right] \text{ and } n=3548\left[\frac{1}{(2.041)^2}-\frac{1}{(m+0.5)^2}\right]$$

with $m=3, 4, 5\ldots\ldots$ The deviation of any observed value from the corresponding computed value is shown in Column IV. Beyond the maximum at $734''$ all further estimates are somewhat open to doubt on account of crowding of maxima.

It will be seen that each group, except that of $663''$, is represented up to the region where the difference between successive computed values is of the same order of magnitude as the confusion mentioned above. It should be noted that, between the last computed maximum listed in the table and the converging limit of the series at $n=852''$, there is no pronounced maximum but instead a fairly even distribution of the asteroids, with here and there maxima prominent enough to be counted in any consideration of

weak maxima, as shown by the groups 813″, 825″, and 840″. There is, of course, an infinite number of computed maxima in such a converging region. The number of asteroids in this region is only about a hundred. Therefore there would not be the same possibility of representation as exists in the tabulated region. The maximum at 663″, which is not represented as part of the principal series, may be considered one of a number of minor maxima of whose existence in the main many readers might be skeptical. Most of such minor maxima have been represented by other Rydberg series. More may be said concerning them in a later paper. At first sight the maximum at 763″ may seem as strong as those at 711″ and 734″. If, however, each maximum be divided by the average number of asteroids per second throughout the whole of the immediate group, the two last maxima are seen to be each nearly as important as that at 777″, while that at 663″ is much weaker.

If it be granted that the agreement between the groups of asteroids and the positions calculated from the Rydberg formula be real, we are given thereby a second feature of similarity between atoms and orbital systems. The proof given by Rutherford,[1] by Geiger and Marsden,[2] and by Darwin,[3] from observations on the scattering of alpha particles, that the inverse-square law of attraction holds for interatomic distances, makes it probable that atomic orbits obey the same laws as do those of astronomical systems. The second point of similarity arises from the fact that the Rydberg formula, which was set up empirically to represent spectral series (very probably phenomena of atomic orbits), represents also certain phenomena of astronomical orbits.

While the similarity here suggested between narrow spectral lines and asteroid groups cannot be considered too striking in view of the great breadth of the groups, yet an explanation of the differences may possibly be found in the widely differing frequencies of atomic orbits and planetary orbits. Considering the fact that the frequencies of the vibrations of light are of the order of 10^{15} per

[1] *Philosophical Magazine* (6), **21**, 669–688, 1911.

[2] *Ibid.*, **25**, 604–623, 1913. [3] *Ibid.*, **27**, 504–505, 1914.

second (and orbital frequencies are very probably in simple commensurability therewith), one sees that even in a millionth of a second an atomic orbit is described many more times than an asteroid's orbit during the time since the earliest estimate of its origin. It may be that the sharpness of control of atomic orbits is due to this inconceivably frequent repetition of some controlling influence.

UNIVERSITY OF KANSAS
May 8, 1919

Minor Contributions and Notes

MEASUREMENTS ON THE INDEX OF REFRACTION OF AIR

FOR WAVE-LENGTHS FROM 2218 A TO 9000 A[1]

[Editorial Note.—The valuable researches which have been conducted at the Bureau of Standards on the index of refraction of air appear to be of such importance to active workers in spectroscopy that it has seemed desirable that we should not only call them to the attention of our readers but also extract certain of the data for their convenience. The authors have had the opportunity to smooth out certain very small outstanding differences in the last column of the table (pp. 61–64), so that these values may be regarded as superseding those in the original publication. A few other corrections were made by them in May 1919. See also the article by Professor R. T. Birge in this number.

The practical astronomer will find data regarding astronomical refraction in section 4, which might otherwise escape his attention.

Sections 5 and 6 are of obvious interest to physicists.]

I. CORRECTION OF WAVE-LENGTH MEASUREMENTS MADE IN AIR AT OTHER THAN NORMAL TEMPERATURES AND PRESSURES

The international wave-length standards are represented by specified radiations whose wave-lengths were measured in dry air at 15° C., exerting a barometric pressure equivalent to 760 mm of mercury.[2] These standards serve as a basis for the precise measurement of all other wave-lengths. Variations in the density of the air appreciably affect the absolute values of the wave-lengths, and when wave-length measurements are made in air whose temperature is not 15° C., or whose pressure is not equal to 760 mm, corrections must be applied to the measured values to reduce them to their values in air under normal conditions. If the index of refraction of air were constant throughout the spectrum all the wave-lengths would vary in the same ratio, but the dispersion

[1] This paper by W. F. Meggers and C. G. Peters, Associate Physicists, Bureau of Standards, is an extract from *Scientific Papers of the Bureau of Standards*, S. W. Stratton, Director, No. 327. Issued October 31, 1918. Price, 10 cents; sold only by the Superintendent of Documents, Government Printing Office, Washington, D.C.

[2] *Astrophysical Journal*, **32**, 215, 1910; **33**, 85, 1911; **39**, 93, 1914.

of the air makes this variation a function of the absolute index and of the density of the air at the time the wave-length comparisons were made. For example, let a wave-length λ be measured in terms of a standard wave-length λ' and consider the result if the measurement is made in air whose density is greater than that defined by the normal conditions. The indices of refraction n and n' for these two wave-lengths increase proportionately with the density of the air, and the wave-lengths decrease. But if λ is smaller than λ' the absolute index of refraction n is larger than n', and the increase in index due to the increased density of air will be a larger amount for n than for n'. Consequently λ is proportionately shorter than λ' in denser air and requires a positive correction to make it comparable with λ' in air at 15° C. and 760 mm.

The effect of the temperature and pressure of the air must be taken into account in all accurate comparisons of wave-lengths made either by the coincidence method with diffraction gratings[1] or with interferometers.[2] The necessary corrections are generally small and may be negligible under certain conditions. In order actually to calculate these corrections let us imagine the wave-length comparisons to be made by means of an interferometer in air whose density d is defined by the temperature t and the barometric pressure B. Let λ represent the wave-length of a radiation under these conditions, n the index of refraction of this air for this wave-length, and λ' and n' analogous quantities for the standard wave-length with which λ is to be compared. The same letters with the subscript o may be used to represent the same quantities under standard observing conditions; that is, air at 15° C. and 760 mm. It is desired to obtain λ_o. The orders of interference measured under the actual conditions give $p\lambda = p'\lambda'$ $(= 2e)$. The calculation for λ, however, is made from $\frac{p'\lambda_o'}{p}$ in which the standard wave-length is used as if the air had no dispersion, and then a correction is applied.

[1] Kayser, *Handbuch der Spectroscopie*, 1, 719, 1900.

[2] Buisson and Fabry, *Journal de Physique*, 7, 169, 1908.

This correction, the difference between the exact value and that calculated, is equal to

$$\delta=\lambda_0-\frac{p'\lambda_0'}{p}=\lambda_0\left(1-\frac{p'\lambda_0'}{p\lambda_0}\right)=\lambda_0\left(1-\frac{\lambda\lambda_0'}{\lambda'\lambda_0}\right).$$

But

$$\frac{\lambda}{\lambda_0}=\frac{n_0}{n} \text{ and } \frac{\lambda'}{\lambda_0'}=\frac{n_0'}{n'}.$$

Then

$$\delta=\lambda_0\left(1-\frac{n_0n'}{n_0'n}\right)=\lambda_0\frac{n_0'n-n_0n'}{n_0'n}=\lambda_0(n_0'n-n_0n'),$$

since $n_0'n$ in the denominator may be called unity. Assuming the refractivity to be proportional to the density,

$$\frac{n-1}{d}=\frac{n_0-1}{d_0} \text{ and } \frac{n'-1}{d}=\frac{n_0'-1}{d_0},$$

we have

$$n=\frac{(n_0-1)d}{d_0}+1 \text{ and } n'=\frac{(n_0'-1)d}{d_0}+1,$$

and the correction reduces to

$$\delta=\lambda_0(n_0-n_0')\frac{d-d_0}{d_0}.$$

The factor $\frac{d-d_0}{d_0}$ is easily calculated as a function of temperature and pressure, and is constant for all the waves compared under these conditions.

The quantity $\lambda_0\,(n_0-n_0')$ may be calculated as a function of the wave-length and represented by a curve from which the correction to the wave-length can be obtained by multiplying its ordinate by the appropriate density factor $\frac{d-d_0}{d_0}$.[1]

2. CORRECTIONS FOR WATER-VAPOR IN THE AIR

The refractive index of water-vapor has not been fully investigated, and the corrections necessary to change wave-lengths measured in wet air to their values in dry air are therefore not very

[1] To avoid all these troublesome computations, tables have been prepared and reproduced in the *Bulletin of the Bureau of Standards* to give these corrections for the entire range of wave-lengths and densities of air in which accurate spectroscopic measurements are ordinarily made.

accurately known. Lorenz measured the index of refraction of water-vapor (specific gravity = 0.0008061) for sodium light as 1.0002500. From this result he recommends a correction of $+0.000041\frac{m}{760}$ to indices of refraction measured in air containing water-vapor of m millimeters pressure.

No measurements on the dispersion of water-vapor are known to exist, but if the dispersion is assumed to be comparable with that of air the corrections to relative wave-lengths will not be affected, since they depend on the difference of indices $n_0 - n_0'$.

Two of our experiments on the dispersion of air for waves between 5800 A and 7500 A were made with air having a water-vapor pressure of 13 mm. The index of refraction was observed to be diminished by approximately the amount given by the Lorenz expression. Furthermore, this decrease was about the same for different wave-lengths. These results on wet air are not complete enough to add much to the subject of dispersion of light by water-vapor, but they show that the corrections to index of refraction as recommended above for water-vapor in the air are quite proper.

3. CORRECTIONS TO CONVERT TO VACUUM VALUES

Corrections required to convert wave-lengths or oscillation frequencies measured in air at 20° C. and 760 mm to their value in vacuum are found in Watts's *Index of Spectra*, Appendix E (p. 51), and in Kayser's *Handbuch der Spectroscopie*, **2**, 513. These are applicable to Rowland's standard wave-lengths and all wave-lengths determined from them, but must be revised to apply to the international system of wave-lengths, which represents values measured in air at 15° C. and 760 mm.

The dispersion equation representing our observations on the indices of refraction of air at 15° C. and 760 mm was used to construct a new table of corrections. The table (p. 61) gives the values of wave-length in air (λ), refractivity $(n-1)10^7$, vacuum correction to λ ($n\lambda - \lambda$), oscillation frequency in air $\left(\frac{1}{\lambda}\right)$, and vacuum correction to frequency $\left(\frac{1}{n\lambda} - \frac{1}{\lambda}\right)$ from 2000 A to 10,000 A at intervals

of 50 A. This table shows, for example, that a light-wave of length 5000.000 A in dry air at 15° C. and 760 mm is increased in length by 1.391 A in a vacuum. The frequency (number of waves per centimeter) in air is 20,000.00, but in a vacuum it is diminished by 5.56. The vacuum frequencies corresponding to any wave-lengths in the international scale can be readily determined by the use of the table (p. 61) together with a table of reciprocals with seven-place arguments.

4. ASTRONOMICAL REFRACTION

Newcomb states in his *Compendium of Spherical Astronomy* (p. 223):

> There is, perhaps, no branch of practical astronomy on which so much has been written as on this (astronomical refraction) and which is still in so unsatisfactory a state. The difficulties connected with it are both theoretical and practical. The theoretical difficulties arise from the uncertainty and variability of the law of diminution of the density of the atmosphere with height, and also from the mathematical difficulty of integrating the equations of the refraction for altitudes near the horizon, after the best law of diminution has been adopted.

The practical difficulties involve the relation of refractive index with density of air as depending upon pressure, temperature, and humidity, and on the necessary inattention to the dispersion of light. In practice the astronomer usually disregards all dependence of refractive index on color. This must involve errors in zenith-distance observations on stars whose colors are intrinsically different and especially in observations of stars near the horizon, where the short waves are largely absorbed and scattered by our atmosphere, causing the stars to be represented only by longer waves. Because of these difficulties tables of corrections for astronomical refraction are probably better determined from astronomical observations than from laboratory measures. The refraction tables which have been applied to the major portion of astronomical observations are those of Bessel, founded upon the observations of Bradley and published in *Tabulae Regiomontanae*. In 1870 the Pulkowa tables were published under the title *Tabulae Refractionium in usum speculae Pulcovensis Congestae*. These

TABLE OF CORRECTIONS TO BE APPLIED TO WAVE-LENGTHS IN AIR AT 15° C. AND 760 MM TO REDUCE WAVE-LENGTHS AND FREQUENCIES TO VACUUM VALUES

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ Add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ Subtract
2000	3255.82	0.6512	50,000	16.274
2050	3220.12	.6601	48,780	15.703
2100	3187.86	.6695	47,619	15.176
2150	3158.63	.6791	46,511	14.687
2200	3132.07	.6891	45,454	14.232
2250	3107.87	.6993	44,444	13.809
2300	3085.75	.7097	43,478	13.412
2350	3065.50	.7204	42,553	13.041
2400	3046.91	.7313	41,666	12.692
2450	3029.81	.7423	40,816	12.363
2500	3014.05	.7535	40,000	12.053
2550	2999.48	.7649	39,215	11.759
2600	2986.00	.7764	38,461	11.481
2650	2973.50	.7880	37,735	11.217
2700	2961.88	.7997	37,037	10.967
2750	2951.08	.8115	36,363	10.728
2800	2941.00	.8235	35,714	10.500
2850	2931.60	.8355	35,087	10.283
2900	2922.80	.8476	34,482	10.076
2950	2914.57	.8598	33,898	9.877
3000	2906.85	.8721	33,333	9.687
3050	2899.60	.8844	32,786	9.504
3100	2892.79	.8968	32,258	9.329
3150	2886.38	.9092	31,746	9.160
3200	2880.33	.9217	31,250	8.998
3250	2874.63	.9343	30,769	8.842
3300	2869.24	.9469	30,303	8.692
3350	2864.15	.9595	29,850	8.547
3400	2859.33	.9722	29,411	8.407
3450	2854.76	.9849	28,985	8.272
3500	2850.43	.9977	28,571	8.142
3550	2846.32	1.0104	28,169	8.016
3600	2842.41	1.0233	27,777	7.893
3650	2838.69	1.0361	27,397	7.774
3700	2835.16	1.0490	27,027	7.660
3750	2831.79	1.0619	26,666	7.550
3800	2828.58	1.0749	26,315	7.442
3850	2825.51	1.0878	25,974	7.337
3900	2822.59	1.1008	25,641	7.235
3950	2819.79	1.1138	25,316	7.137

TABLE OF CORRECTIONS—*Continued*

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ Add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ Subtract
4000	2817.12	1.1268	25,000	7.041
4050	2814.56	1.1399	24,691	6.948
4100	2812.11	1.1530	24,390	6.857
4150	2809.76	1.1661	24,096	6.769
4200	2807.51	1.1792	23,809	6.683
4250	2805.36	1.1923	23,529	6.599
4300	2803.29	1.2054	23,255	6.517
4350	2801.30	1.2186	22,988	6.437
4400	2799.39	1.2317	22,727	6.360
4450	2797.55	1.2449	22,471	6.285
4500	2795.78	1.2581	22,222	6.211
4550	2794.08	1.2713	21,978	6.139
4600	2792.44	1.2845	21,739	6.069
4650	2790.86	1.2978	21,505	6.000
4700	2789.34	1.3110	21,276	5.933
4750	2787.88	1.3242	21,052	5.868
4800	2786.46	1.3375	20,833	5.804
4850	2785.09	1.3508	20,618	5.741
4900	2783.78	1.3640	20,406	5.680
4950	2782.50	1.3773	20,202	5.620
5000	2781.27	1.3906	20,000	5.561
5050	2780.08	1.4039	19,801	5.503
5100	2778.93	1.4173	19,607	5.447
5150	2777.81	1.4306	19,417	5.392
5200	2776.74	1.4439	19,230	5.338
5250	2775.69	1.4572	19,047	5.286
5300	2774.68	1.4706	18,867	5.234
5350	2773.70	1.4839	18,691	5.183
5400	2772.75	1.4973	18,518	5.133
5450	2771.83	1.5106	18,348	5.085
5500	2770.94	1.5240	18,181	5.037
5550	2770.07	1.5374	18,018	4.990
5600	2769.23	1.5508	17,857	4.944
5650	2768.41	1.5642	17,699	4.899
5700	2767.62	1.5775	17,543	4.854
5750	2766.85	1.5909	17,391	4.811
5800	2766.10	1.6043	17,241	4.768
5850	2765.37	1.6177	17,094	4.726
5900	2764.66	1.6311	16,949	4.685
5950	2763.98	1.6446	16,806	4.644
6000	2763.31	1.6580	16,666	4.604
6050	2762.65	1.6714	16,528	4.565
6100	2762.02	1.6848	16,393	4.527
6150	2761.40	1.6983	16,260	4.489
6200	2760.80	1.7117	16,129	4.452

TABLE OF CORRECTIONS—*Continued*

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ Add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ Subtract
6250	2760.22	1.7251	16,000	4.415
6300	2759.65	1.7386	15,873	4.379
6350	2759.09	1.7520	15,748	4.344
6400	2758.55	1.7655	15,625	4.309
6450	2758.02	1.7789	15,503	4.275
6500	2757.51	1.7924	15,384	4.241
6550	2757.00	1.8058	15,267	4.208
6600	2756.51	1.8193	15,151	4.175
6650	2756.04	1.8328	15,037	4.143
6700	2755.57	1.8462	14,925	4.112
6750	2755.11	1.8597	14,814	4.081
6800	2754.67	1.8732	14,705	4.050
6850	2754.23	1.8866	14,598	4.020
6900	2753.81	1.9001	14,492	3.990
6950	2753.39	1.9136	14,388	3.961
7000	2752.99	1.9271	14,285	3.932
7050	2752.59	1.9406	14,184	3.903
7100	2752.21	1.9541	14,084	3.875
7150	2751.83	1.9676	13,986	3.847
7200	2751.46	1.9811	13,888	3.820
7250	2751.10	1.9945	13,793	3.793
7300	2750.74	2.0080	13,698	3.767
7350	2750.39	2.0215	13,605	3.741
7400	2750.06	2.0350	13,513	3.715
7450	2749.72	2.0485	13,422	3.690
7500	2749.40	2.0620	13,333	3.665
7550	2749.08	2.0756	13,245	3.640
7600	2748.77	2.0891	13,157	3.616
7650	2748.47	2.1026	13,071	3.592
7700	2748.17	2.1161	12,987	3.568
7750	2747.88	2.1296	12,903	3.545
7800	2747.59	2.1431	12,820	3.522
7850	2747.31	2.1566	12,738	3.499
7900	2747.03	2.1702	12,658	3.476
7950	2746.76	2.1837	12,578	3.454
8000	2746.50	2.1972	12,500	3.432
8050	2746.24	2.2107	12,422	3.410
8100	2745.99	2.2243	12,345	3.389
8150	2745.74	2.2378	12,269	3.368
8200	2745.49	2.2513	12,195	3.347
8250	2745.25	2.2648	12,121	3.326
8300	2745.02	2.2784	12,048	3.306
8350	2744.79	2.2919	11,976	3.286
8400	2744.56	2.3054	11,904	3.266
8450	2744.34	2.3190	11,834	3.246

TABLE OF CORRECTIONS—*Continued*

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ Add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ Subtract
8500	2744.12	2.3325	11,764	3.227
8550	2743.90	2.3460	11,695	3.208
8600	2743.69	2.3596	11,627	3.189
8650	2743.49	2.3731	11,560	3.170
8700	2743.29	2.3867	11,494	3.152
8750	2743.09	2.4002	11,428	3.134
8800	2742.89	2.4137	11,363	3.116
8850	2742.70	2.4273	11,299	3.099
8900	2742.51	2.4408	11,235	3.081
8950	2742.32	2.4544	11,173	3.063
9000	2742.14	2.4679	11,111	3.046
9050	2741.96	2.4815	11,049	3.029
9100	2741.79	2.4950	10,989	3.012
9150	2741.61	2.5086	10,928	2.995
9200	2741.44	2.5221	10,869	2.979
9250	2741.28	2.5357	10,810	2.963
9300	2741.11	2.5492	10,752	2.947
9350	2740.95	2.5628	10,695	2.931
9400	2740.79	2.5763	10,638	2.915
9450	2740.64	2.5899	10,582	2.899
9500	2740.48	2.6035	10,526	2.884
9550	2740.33	2.6170	10,471	2.869
9600	2740.18	2.6306	10,416	2.854
9650	2740.04	2.6441	10,362	2.839
9700	2739.89	2.6577	10,309	2.824
9750	2739.75	2.6713	10,256	2.809
9800	2739.61	2.6848	10,204	2.795
9850	2739.47	2.6984	10,152	2.780
9900	2739.34	2.7119	10,101	2.766
9950	2739.20	2.7255	10,050	2.752
10,000	2739.07	2.7391	10,000	2.738

are based upon the researches of Gylden[1] and are supplemented by those of Fuss.[2] The Pulkowa tables are founded upon a smaller refraction constant than Bessel's, and Newcomb (*loc. cit.*) says the most recent discussions and comparisons indicate a still greater diminution in the constant. The constants under discussion represent the index of refraction of air at 0° C., 760 mm, for white light as follows.[3]

[1] *Mem. de l'Acad. de St. Petersburg*, **10**, No. 1, 1866, and **12**, No. 4, 1868.

[2] *Ibid.*, **18**, No. 4, 1868. [3] Winkelmann, *Handbuch der Physik* (2), **6**, 534.

Bessel	1.00029257, from Bradley.
Gylden	29232, from Pulkowa, 1842–1849
Fuss	29161, from Pulkowa, 1867–1869
Newcomb	29195, from Greenwich, 1877–1886
Bauschinger	29152, from Munich, 1891–1893
Courvoisier	29180, from Heidelberg, 1900–1903

Under the same conditions our observations give about 1.0002923, assuming the wave-length 5560 A of maximum visibility for white light.

For photographic work a larger refraction constant must be used than for visual. If the photographic maximum is at 4200 A the refraction constant should be 1.0002968 and the photographic corrections are therefore 1.0154 larger than the visual. The ratio of these corrections has been found experimentally by Wilsing[1] to be 1.01539.

5. OPTICAL TEMPERATURE-COEFFICIENT

If the Dale and Gladstone law, $\frac{n-1}{d}=\text{constant}$, holds for air, the temperature-coefficient of index-variation should be identical with the temperature-coefficient of volume-change at constant pressure and any refractivity of air at temperature $t°$ C. could be expressed in terms of the refractivity of air at 0° C. by the relation

$$(n-1)_t=\frac{(n-1)_0}{1+at},$$

in which $a=0.00367$.

The following values of the optical temperature-coefficient of air show considerable disagreement: Lorenz, 0.00367; von Lang, 0.00310; Mascart, 0.00383; Benoit, 0.00367; and Walker, 0.00360.

Our observations on the refractivity of air at temperatures of 0, 15, and 30° C. show that a is a function of the wave-length and increases rapidly as the absorption band in the ultra-violet is approached. This is shown in the following table.

[1] *Astronomische Nachrichten*, **145**, 293, 1898.

λ	a
8500	0.003672
7500	3674
6500	3678
5500	3685
4500	3700
3500	3738
2500	3872

A decrease in the density of air due to an increase in its temperature either apparently or actually changes the position of the ultra-violet absorption band. As the temperature of the air rises the absorption and dispersion of short waves of light diminish, and this effect is probably explained by the increased kinetic energy of the gas molecules rather than by the decreased density of the gas. It would be interesting to test this by ultra-violet dispersion measurements of air with density reductions at constant temperature.

Within the range of our observations the refractivity of air at any temperature for any particular wave-length λ can be obtained from measurements on air at 0° C. by the relation mentioned above if a correction be applied to a, which is a function of λ. A very simple correction to a is contained in the expression

$$(n-1)_t=\frac{(n-1)_0}{1+t\left(a+\frac{3\times 10^6}{\lambda^3}\right)}$$

in which $a=0.00367$, which allows the calculation of $(n-1)_t$ within the limits of probable error of observation.

6. DISPERSION FORMULAE

A large amount of theoretical work on refraction and dispersion of transparent materials has been done in the past, and various theories have been tested by measurements of indices of refraction of solids and liquids over a wide range of spectrum from the ultra-violet through the visible and far out into the infra-red. It is especially important for this purpose to measure indices of refraction in the neighborhood of absorption bands. Measurements of refractivities of gases have heretofore been confined to a compara-

tively small portion of the spectrum, and although more work has been done with air than with other gases the review of previous work showed how incomplete it was. It was hoped that the present investigation of the dispersion of air for red and infra-red light would give an indication of an absorption band in the infra-red so that the Sellmeier or Ketteler-Helmholtz dispersion formula could be tested in its representation of the observations. The empirical Cauchy equation with three constants was first used to represent the measurements, and although the representation was quite satisfactory there seemed to be a slight deviation in the red which might be explained by an absorption band in the infra-red. An effort was made to locate such a band from our dispersion measurements, but without success.

Water-vapor has a very complicated absorption spectrum in the infra-red, but dry air is known to be quite transparent to long waves. The only constituent of air which seems to absorb much infra-red light is carbon dioxide. Although carbon dioxide constitutes only 0.03 per cent of the volume of ordinary air, the absorption is quite marked for wave-lengths 42,700 A[1] and 147,000 A[2] when these waves are received through several meters of air. Rubens and Wartenburg[3] have found dry air to be quite transparent for wave-lengths 230,000 A, 520,000 A, 1,100,000 A, and 3,140,000 A. The absorption bands due to the carbon dioxide in the air are narrow and weak compared to the absorption band of air for ultra-violet light, and it seems doubtful that they can exert any marked influence on dispersion. The effect of these bands is probably similar to that of the *A* band due to atmospheric oxygen which strongly absorbs light of wave-length 7600 A, but relatively small thicknesses of air freely transmit these waves.

Koch[4] measured the index of refraction of air as 1.00028806 for residual rays from gypsum ($\lambda = 67{,}094$ A) and 1.00028875 for residual rays from calcite ($\lambda = 86{,}784$ A). If no absorption band

[1] Paschen, *Wiedemanns Annalen*, **53**, 334, 1894; Statescu, *Philosophical Magazine*, **30**, 737, 1915.

[2] Rubens, *Wiedemanns Annalen*, **64**, 584, 1898.

[3] *Physikalische Zeitschrift*, **12**, 1080, 1911.

[4] *Nova Acta Soc. Upsala* (4), **2**, 1909.

exists between 9000 A and 67,000 A, and if it is permissible to extrapolate the formula representing our observations, we obtain $n=1.0002876$ for the longest waves.

It is probably necessary, therefore, to conclude that if there is any marked absorption of infra-red waves by air, the band of absorption must lie at extremely long waves.

Moreover, the electromagnetic theory has connected the index of refraction n of a medium to its dielectric constant D by the relation $n^2=D$. This relation was derived for long waves and is not generally satisfied in the visible spectrum, where all substances have more or less dispersion. For certain gases, such as hydrogen, nitrogen, and dry air, which show only small dispersion and no strong absorption in the infra-red, $n^2=D$ approximately, even when n is taken for waves in the visible spectrum.[1] For example,[2] D for dry air is 1.000590 and n^2 for sodium light is 1.000584. The dispersion for such substances is well represented by the simple empirical formula of Cauchy

$$n=a+\frac{b}{\lambda^2}+\frac{c}{\lambda^4}.$$

If n^2 is not equal to D for visible rays, the dispersion cannot be represented by Cauchy's formula, but the Sellmeier or Ketteler-Helmholtz formula, which takes into account the free periods corresponding to absorption bands of wave-length λ_κ, must be used. Neglecting absorption for wave-lengths λ, this formula is generally written as follows:

$$n^2=1+\Sigma\frac{m_\kappa\lambda^2}{\lambda^2-\lambda_\kappa^2}.$$

The effect on n of each species of ions K is represented by a term in the sum Σ in which m_κ is the dielectric constant of the K ions. Then $1+\Sigma m_\kappa$ is the observed dielectric constant D of the substance for infinitely long waves. If there are two families of resonance ions, one producing the ultra-violet absorption band of wave-length

[1] Tangle, *Annalen der Physik* (4), **26**, 59, 1908.

[2] Schmidt, *Annalen der Physik* (4), **11**, 121, 1903.

λ_v and the other responsible for the infra-red absorption band of wave-length λ_r, the dispersion is represented by

$$n^2 = 1 + \frac{m_v\lambda^2}{\lambda^2 - \lambda_v^2} + \frac{m_r\lambda^2}{\lambda^2 - \lambda_r^2}.$$

Miss Howell[1] gave a Sellmeier formula for air having an infra-red absorption band of wave-length 31,800 A, "probably due to traces of water vapor," but this seems very doubtful.

If there is only one family of resonance ions, the formula reduces to

$$n^2 = 1 + \frac{m_v\lambda^2}{\lambda^2 - \lambda_v^2}$$

and this first term of Sellmeier's formula gives a satisfactory representation of the dispersion of all transparent substances in which the infra-red absorption is negligible.

The observational equations previously described were solved by the method of least squares and gave the following expression for the dispersion of air at 0° C. and 760 mm:

$$n^2 = 1 + \frac{0.0005737\,8\lambda^2}{\lambda^2 - 595{,}260}.$$

This simple Sellmeier dispersion formula represents the observations nearly as well as the Cauchy formula given above.

7. SUMMARY

A survey of previous researches on refraction of air shows that many investigators have worked either with white light or with one monochromatic radiation, and dispersion measurements have been limited to a small interval of the spectrum. No index measurements exist for waves longer than those corresponding to orange light, and in the ultra-violet the dispersion formulae disagree by more than 10 per cent of the refractivity.

Recent work in spectroscopy makes it very desirable to have more accurate and extensive data on the index of refraction and

[1] *Physical Review*, **6**, 81, 1915.

dispersion of air. The international system of standard wave-lengths expresses the lengths of waves in air at 15° C. and 760 mm, and all wave-length measurements made under other conditions require small corrections because of the effect of temperature and pressure of the air upon its optical dispersion. Furthermore, it is often desirable to multiply wave-lengths measured in air by the indices of refraction of air for these wave-lengths and thus convert them to their value in a vacuum. An accuracy of one part in several millions is now striven for in the measurement of wave-lengths, and to maintain their relative accuracy in the values reduced to vacuum it is necessary to know the indices of refraction within a few units in the seventh decimal place.

For several years this Bureau has been engaged in the accurate measurement of wave-lengths. Interferometer comparisons of standard wave-lengths have been made throughout a large range of spectrum and the grating spectra of more than 50 of the chemical elements have been photographed and measured in the red and infra-red spectral regions. In connection with these accurate measurements of wave-lengths it was thought advisable to measure the absolute indices of refraction of air for the entire spectral region which is accessible to photography.

Accuracy and efficiency recommended the use of the Fabry and Perot interferometer for this work, since this apparatus can be conveniently inclosed in a chamber in which the temperature and pressure of the air can be regulated as desired, and it also permits simultaneous observations for a large number of different wave-lengths. Sections of the circular fringes, produced by various radiations from a source of light illuminating the parallel plates of the interferometer, were photographed either with a grating or a rock-salt prism spectrograph, first when the space between the plates was evacuated, and then when dry air at measured temperature and pressure was present.

The index of refraction for a particular wave-length was obtained directly from measurements of the photographed interference fringes which allowed the ratio of lengths of this wave in vacuum and in air to be calculated. Observations were made at spectrum

intervals of about 40 A from the extreme ultra-violet at 2200 A, through the visible spectrum, and in the infra-red to 9000 A.

Complete sets of observations were made on dry air at atmospheric pressure and at temperatures of 0, 15, and 30° C. These are quite closely represented by the following dispersion formulae of the Cauchy form:

$$(n-1)_0 \times 10^7 = 2875.66 + \frac{13.412}{\lambda^2 \times 10^{-8}} + \frac{0.3777}{\lambda^4 \times 10^{-16}}$$

$$(n-1)_{15} \times 10^7 = 2726.43 + \frac{12.288}{\lambda^2 \times 10^{-8}} + \frac{0.3555}{\lambda^4 \times 10^{-16}}$$

$$(n-1)_{30} \times 10^7 = 2589.72 + \frac{12.259}{\lambda^2 \times 10^{-8}} + \frac{0.2576}{\lambda^4 \times 10^{-16}}.$$

These observations are used in the construction of a table giving the corrections which must be applied to wave-lengths measured in air whose density is not normal. A table of corrections to convert wave-lengths or frequencies measured in air to their values in a vacuum is also given.

The coefficient of index variation with temperature was found from these measurements to be a function of the wave-length. For long waves this optical temperature-coefficient is identical with the density temperature-coefficient, that is, $\frac{1}{273}$, but as the ultra-violet absorption band is approached it increased rapidly, becoming $\frac{1}{258}$ at 2500 A.

There seems to be no definite evidence of any strong absorption of infra-red light by dry air, and it is therefore possible to represent the optical dispersion of air by the first term of Sellmeier's formula quite satisfactorily.

W. F. MEGGERS
C. G. PETERS

WASHINGTON, D.C.
March 13, 1918

THE DISPERSION OF AIR AND THE REDUCTION OF WAVE-LENGTHS TO VACUUM

As long as the wave-lengths of light were founded on the Rowland system it was quite customary, in reducing to vacuum, to use the table printed in Kayser's *Handbuch* (**2**, 514). Since the new International System (I.A.)[1] has come into general use, there has been no uniformity in this reduction. This has led to different values of the frequency in vacuum, even when founded on the same experimental data. Moreover, in some cases the reduction has been made quite incorrectly. There have been carried out, within recent years, several new investigations on the index of refraction of air, thus giving a new chance for variation in the reduction. Finally, there are a number of apparent inconsistencies in the published work relating to this matter. These have caused the author considerable confusion, and may have affected others similarly.

It thus seems advisable to agree on one method for the reduction to vacuum, and a method is here tentatively suggested, with a table of values based upon it. It is perhaps needless to remark that as far as spectral laws are concerned any consistent method of reduction which is not woefully in error would be satisfactory. It is confusing, however, in trying to correlate various spectral formulae, or in checking the data of others, to have in use so many different systems of reduction to vacuum.

The Rowland[2] system of wave-lengths is founded upon the wave-length of the sodium lines in ordinary air at about 20°C. and 760 mm pressure. The International System resulted from Michelson's[3] determination of the wave-length of the red cadmium line (6438.4722 I.A.) in ordinary air at 15°C. (mercury-in-glass thermometer) and 760 mm pressure. In this work no accurate record of the humidity was kept. The experiment was repeated by Benoit, Fabry, and Perot,[4] and this time the humidity was

[1] See *Transactions of the International Solar Union*, **3**, 135, 1910.

[2] *Astronomy and Astrophysics*, **12**, 323, 1893.

[3] *Travaux et Mémoires du Bureau International des Poids et Mesures*, **11**, 1, 1895.

[4] *Ibid.*, **15**, 3, 1913. See also *Transactions of the International Solar Union*, **2**, 109–137, 1907.

carefully considered. The final result was 6438.4696 I.A., reduced to dry air at 15°C. (hydrogen thermometer) and 760 mm pressure. Michelson's determination, as reduced by the foregoing authors to the hydrogen thermometer and to dry air, is identical with their value, if a humidity of 68 per cent be assumed for Michelson's work. The final value of Benoit, Fabry, and Perot has been adopted as the definition of the I.A., giving

$$\text{I.A.} = \frac{\text{wave-length of red Cd line, in dry air at 15°, 760 mm, with } g = 980.67}{6438.4696}$$

The first really accurate measurements of the dispersion-curve for air were made by Kayser and Runge.[1] They used ordinary air and derived as the least-squares solution of the experimental data, in terms of the usual (empirical) Cauchy formula,

$$\mu - 1 = 0.00028787 + \frac{13.16}{\lambda^2 10^{15}} + \frac{3.16}{\lambda^4 10^{24}}, \qquad (1)$$

where λ is in cm, and μ applies to 0°C., 760 mm pressure. A more customary manner of writing this is

$$10^7(\mu - 1) = 2878.7 + \frac{13.16}{\lambda^2} + \frac{0.316}{\lambda^4}, \qquad (2)$$

where λ is in microns.

Kayser[2] in quoting (2) states that λ is in $\mu\mu$. This error has been repeated in Baly's *Spectroscopy* (p. 583), while on page 146 of the same text the unit is given correctly.

In $\mu\mu$ we would have

$$\mu - 1 = 0.00028787 + \frac{1.316}{\lambda^2} + \frac{31600}{\lambda^4}, \qquad (3)$$

a form of the equation used by some investigators.

The most preferable form for modern work, however, is that in which λ is expressed in angstroms. This form is

$$10^7(\mu - 1) = 2878.7 + \frac{13.16}{\lambda^2 . 10^{-8}} + \frac{0.316}{\lambda^4 . 10^{-16}}. \qquad (3')$$

Now (1) applies to ordinary air at 0°C. and 760 mm pressure. Kayser and Runge (*loc. cit.*) estimated that the vapor-pressure

[1] *Wiedemanns Annalen*, **50**, 293–315, 1893. [2] *Handbuch*, **2**, 513.

under the experimental conditions was 5 to 7 mm, and that this would decrease the index about 3×10^{-7}. Therefore for *dry* air we have

$$10^7(\mu-1)=2881.7+\frac{13.16}{\lambda^2}+\frac{0.316}{\lambda^4}, \qquad (4)'$$

where λ is in microns.

The original experimental data given by Kayser and Runge (*loc. cit.*) apply to formula (2). The same experimental results, however, corrected by 3×10^{-7}, are given by Runge,[1] together with formula (4) and a brief table of corrections to vacuum. The table of corrections given in Kayser's *Handbuch* (**2**, 514) is based on formula (4), while formula (2) accompanies the table and supposedly applies to it. The values given in the *Handbuch* (**1**, 719) are based on formula (2). Thus the corrections to vacuum that nearly all investigators have used (table, *Handbuch*, **2**, 514) are based on Kayser and Runge's determination of the index of refraction for dry air, while the index for ordinary air should really have been used. The difference ($3\times10^{-7}\lambda$) varies from 0.002 to 0.001 A in the visible spectrum. This difference is immaterial, except for the confusion resulting from the apparent inconsistency in the published data.

Since 1893 a number of investigations of the dispersion of air have been made, the most important being by Rentschler,[2] Cuthbertson,[3] Howell,[4] and Dickey.[5] Some investigators, like Cuthbertson, used dry air and quote Kayser's formula (4) correctly as for dry air. Other observers, from the conditions of the experiment, may have used ordinary air. In comparing the various results, however, no distinction has been made, in some cases, between dry and ordinary air. Thus Miss Howell includes Cuthbertson's formula for dry air, Kayser and Runge's for ordinary air, and Rentschler's (dry air) with her own. The objection to this is that the difference (3×10^{-7}) is of the same order of magnitude as the experimental errors.

[1] *Astronomy and Astrophysics*, **12**, 426, 1893.

[2] *Astrophysical Journal*, **28**, 345, 1908.

[3] *Proc. Roy. Soc.* (A), **83**, 151, 1910.

[4] *Physical Review* (2), **6**, 81, 1915.

[5] *Astrophysical Journal*, **45**, 189, 1917.

Before deciding on the best dispersion-curve to use in the reduction to vacuum it seems pertinent to inquire what values have been used by others. For the Rowland system the actual correction in λ is given by $\frac{273}{293}(\mu-1)_0\times\lambda_{vac}$ or $\frac{273}{293}(\mu-1)_0\times\nu_{vac}$ for the correction in terms of ν. One can of course equally well use λ_{air} and ν_{air} in place of λ_{vac} and ν_{vac}. For the International System the correction should be $\frac{273}{288}(\mu-1)_0\times\lambda_{vac}$, etc.

To make the table of corrections printed in Kayser's *Handbuch* (**2**, 514) available for the I.A. system the printed quantities should be multiplied by $\frac{293}{288}$. This is what Hicks[1] has done, increasing the printed values by $\frac{1}{60}$ of themselves—a close enough approximation. Hicks is here using Wood's[2] experimental data for the principal sodium series. Wood's own reduction to vacuum is incorrect and it is not apparent to the author how it was obtained.

Fowler[3] and Nicholson[4], although the measurements are in I.A., use Kayser's table uncorrected. The error is thus $\frac{1}{60}$ of the reduction-factor and so varies from 0.018 to 0.035 A in the visible spectrum. This, as mentioned, is of no consequence in fitting spectral formulae, but it can only cause confusion and does not seem a wise practice. Curtis[5] also makes this error, and here it *does* make a difference in the results, since it changes the value of the Rydberg Universal Constant N_0. As the author has pointed out in a separate communication in *Science*,[6] the error will not affect the accuracy with which any given formula fits the data, nor will it affect the universal constancy of N_0. The correct value of N_0 from Curtis' data is 109,678.705 instead of 109,679.22. The author[7] has already obtained 109,678.6 by direct conversion from the Rowland system of the best previous measurements. Curtis' work therefore checks with the older measurements much better than he himself

[1] *Astrophysical Journal*, **44**, 230, 1916.
[2] *Ibid.*, **43**, 73, 1916.
[3] *Proc. Roy. Soc.* (A), **91**, 208, 1915.
[4] *Ibid.*, 255, 1915.
[5] *Ibid.*, **90**, 605, 1914.
[6] *Science*, **48**, 47, 1918.
[7] *Astrophysical Journal*, **32**, 114, 1910.

supposed. There are thus being used at least three different methods of reduction to vacuum, only one of which is correct.

The foregoing constitutes the major portion of a paper which was in the printer's hands, and about to be published (July 1918), when the author learned that the Bureau of Standards had, in spite of the war, continued its work[1] on the dispersion of air, and that the results were about to be published. Accordingly the present paper was withheld until now, when it is possible to present it in conjunction with an account of the work of the Bureau.[2] This new work, because of its great accuracy, completely supersedes all former work on this subject. It is therefore unnecessary to give here the table of corrections to vacuum, prepared by the author, nor is it necessary to present the comparative results of previous investigators. The Bureau article gives a complete synopsis of previous work and also an extensive table of corrections to vacuum. This table should be used in all future work when the wave-lengths are measured in I.A.

RAYMOND T. BIRGE

UNIVERSITY OF CALIFORNIA
December 1918

REGARDING THE *BULLETIN ASTRONOMIQUE*

TO ASTRONOMERS AND GEODESISTS:

L'Observatoire de Paris se propose de consacrer désormais son ***Bulletin Astronomique*** à la publication en langue française de très courts résumés des travaux relatifs à l'Astronomie et à la Géodésie. Nous voudrions que ces résumés fussent faits par les auteurs eux-mêmes; nous les publierons en francais mensuellement, immédiatement après les avoir reçus.

Le titre du recueil sera changé et deviendra:

Revue générale des travaux astronomiques publiée par l'Observatoire de Paris.

[1] See *Scientific Papers of the Bureau of Standards*, No. 312, 380, 1918.

[2] *Ibid.*, full paper, No. 327, October 31, 1918.

Vous faciliteriez grandement notre entreprise en nous envoyant, dès la publication de vos mémoires ou de vos ouvrages, un exemplaire de chacun d'eux avec le court résumé ou l'analyse à insérer dans la *Revue générale des travaux astronomiques.*

B. BAILLAUD, *Le Directeur*

OBSERVATOIRE DE PARIS

NEW GENERAL INDEX OF *ASTROPHYSICAL JOURNAL*

It is the hope of the Editors that a General Index to Vols. XXVI to L of the *Astrophysical Journal* may be issued as promptly as possible after the close of the fiftieth volume with the number for December 1919. It is the present plan to follow the style of the General Index, by authors and by subjects, to Vols. I to XXV, compiled by Professor Storrs B. Barrett and published in 1908 by the University of Chicago Press ($1.50 postpaid). The Editors, however, will be very glad to receive from subscribers any criticisms or suggestions which may tend to make the new index more serviceable, if possible, than its predecessor. Such communications may be addressed to the Managing Editor, EDWIN B. FROST, Williams Bay, Wisconsin.

ERRATA

Vol. **49**, No. 3, April 1919, article on:

"The Magnetic Polarity of Sun-Spots," by George E. Hale, Ferdinand Ellerman, S. B. Nicholson, and A. H. Joy:

Page 158, line 17, *for* "R 28" *read* "V 28."
Page 158, line 20, *for* "V 7" *read* "R_7."
Page 163, line 26 (second line from bottom), *for* "over" *read* "of."
Plate VII, facing page 166, line 5 in legend, *for* "Plate IV *i*" *read* "Plate V *i*."

Reviews

An Introductory Treatise on Dynamical Astronomy. By H. C. Plummer. London: University of Cambridge Press, 1918. American agents, G. P. Putnam's Sons, New York. Pp. 343, figs. 7. $5.50.

Professor Plummer's treatise is intended as an introduction to those portions of astronomy which require dynamical treatment. Hence the main chapters of the book are concerned with the astronomical developments of the Newtonian laws of motion and gravitation. But in a volume of this size it is only possible to treat a limited number of the problems which arise, and the author has confined himself mainly to those which concern the motions of the centers of mass of the bodies about one another under their mutual attractions and of the rigid bodies about their centers of mass. A large portion of the work thus deals with the classic problems of two or three bodies, as represented by the planets and satellites of the solar system, and by the motions of the earth and moon about their respective centers of mass. The more modern portions of the book contain two chapters on the methods by which the orbits of double stars and of spectroscopic binaries are obtained.

Professor Plummer assumes a considerable knowledge of analytical and dynamical processes. The proofs are in general developed quite briefly with the essential steps, but we should imagine that the work will be difficult reading for those who have not had a somewhat extensive training in those parts of mathematics which require symbolic manipulation and the physical interpretation of symbolic results. The reader will also require some considerable familiarity with geometrical astronomy in order to understand in detail the bearing of the various conclusions reached. This is more on account of the brevity, which is perhaps a necessary result of the attempt to compress so much into a single volume, than of omission on the author's part. The book will, we believe, be found more useful to a teacher who wishes to use it as a basis for lectures than to a student who is approaching the subject without assistance.

The various phases of the problem of two bodies naturally occupy a considerable portion of the volume. After giving the elliptic expansions

in several forms as well as the theorems necessary to deal with parabolic and hyperbolic orbits, applications are made to the determination of such orbits from observation. Here, as elsewhere, but little effort is made to deal with the details of the methods which are set forth. For these the reader must go to the special volumes or memoirs dealing with the respective subjects. The dynamical portion of all this work is, of course, the same—namely, motion in a conic section. The chief difficulties are observational and geometrical. For comets and asteroids the given data are positions and the masses of the bodies which mainly determine the motion. For double stars the masses and planes of motion are in general unknown, and we are given only a series of projected distances between the bodies, together with the apparent orientation of these distances. For spectroscopic binaries velocities in the line of sight chiefly constitute the known facts, and so on. The results which can be deduced in this way are exact so far as they go, and they furnish certain relations which must be satisfied if the law of gravitation holds in such systems. This part of the work is, of course, quite independent of the information which may be extracted from the rapidly growing body of knowledge deduced from stellar classification and statistics.

For the planetary theory Professor Plummer adopts the method of the variation of the elements. This method was used by Leverrier in his theories of the planets, and it is certainly the most interesting from the point of view of the mathematician and the most useful for a geometrical description of the motions of the bodies of the solar system. For the major planets, however, the direct method of solution as developed by Newcomb has in the past given an easier mode of approach to numerical results except in the case of Jupiter and Saturn, where G. W. Hill used Hansen's method somewhat modified. But the work in all these cases is simple so long as attention is confined to first-order perturbations. For the general treatment of secular perturbations a chapter contains the method developed by Gauss. The least satisfactory part of Professor Plummer's discussion seems to be that which deals with the connection between the ordinary mode of development in the general theory and the stability of the system. Powers of the time in the coefficients of the periodic terms can be avoided as far as a purely formal expansion of the co-ordinates is concerned, in the same way that they are avoided in the lunar theory. But none of these developments are of much value when we come to a consideration of stability. Practically all of them are divergent series and are only available for a limited time or for a limited

degree of approximation. Thus the old theorems concerning the stability of the major axes, eccentricities, and inclinations, as deduced from the terms of the first and second orders, have now little more than a historic and mathematical interest. Their chief value is a formal one in showing the forms of early terms of the series which are used to represent the positions of the bodies.

The classical methods are well adapted to the calculation of the orbits of the major planets, but very considerable changes are necessary if attempts be made to apply them to many of the asteroids. Professor Plummer only touches slightly on them. An unfortunate slip occurs on page 191, where it is stated that commensurability of the mean motions is inconsistent with stability, evidence for the statement being brought from the fact that the asteroids appear to avoid the regions where their mean motions would be commensurable with that of Jupiter. No proof of this has yet been given. On the contrary, all the investigations so far made indicate that such motions are stable and that the apparent gaps must be explained in some other way. When commensurability is closely approached, the type of motion changes: exact commensurability occurs with an oscillation about that motion, generally called a libration. It is true, as Poincaré has shown, that there are certain conditions under which commensurability cannot occur in the form of a periodic solution; in some of these cases, however, it has been shown that libration still exists but that the oscillation cannot be infinitesimal. In this connection the remark on page 243, that no periodic orbits exist about the triangular equilibrium points when the masses fail to satisfy a certain condition, is open to misconception; it should be added that such orbits exist but that they cannot in general be infinitesimal.

In the later chapters Professor Plummer gives a rapid survey of the Euler-Hill-Adams method for developing the lunar theory, as worked out by the writer, and of the motions of the earth and moon about their respective centers of mass. A useful chapter is the last, on numerical computation, in which the formulae of mathematical quadrature and of harmonic analysis are developed.

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